Statistical Parameterization of the Medial Manifold in Tubelike Structures

Satyaj Bhargava[2], John Lorence[2], Benjamin Cohen[2], Isaiah Jefferson III[2], Therese Nneji[2], Anisha Virmani[4], Minjie Wu[2,3], Howard Aizenstein[2,3], George Stetten[1,2].

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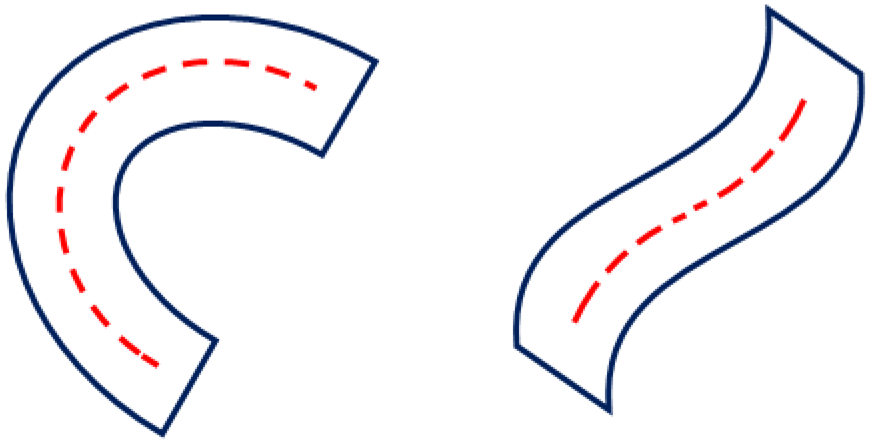
BACKGROUND: MEDIAL MANIFOLDS

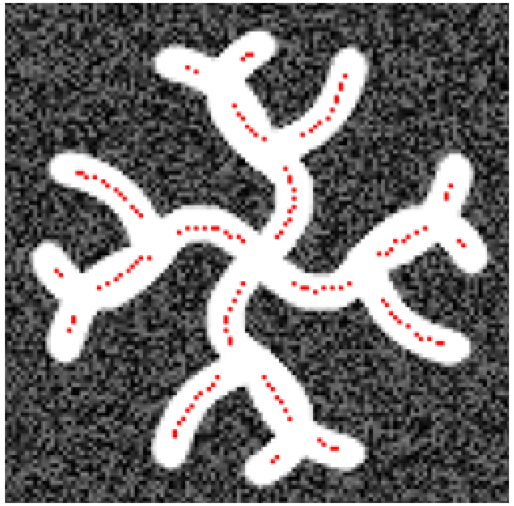
- Medial axis = central line running through a shape or structure.
- Captures geometric and topological properties (e.g., centerlines of vessels).
- Widely used in shape analysis in biomedical imaging.
- Enables measurement of vessel features such as **diameter, curvature, and length.**

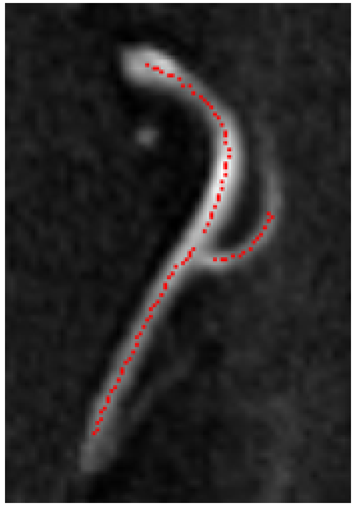


- **Challenge**: extracting clean, accurate medial axes from noisy biomedical images.

STATISTICAL PARAMETERIZATION

- We propose a new technique to obtain the medial manifold in a statistically principled way.
- Using principal component analysis (PCA) to robustly identify medial axes.
- Providing a framework for statistical parameterization of tubelike structures.





- Enabling more reliable, quantitative measurements in disease research, longitudinal studies, and comparative anatomy.

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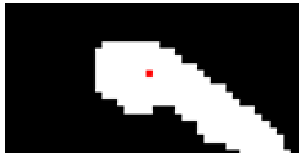
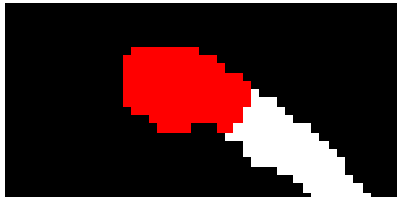
1. Initialization

- Start with a seed voxel inside the segmented tube structure.



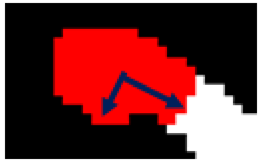
2. Accretion

- Add the voxel nearest to the current center of mass.
- Recalculate the center at each step.
- The set expands symmetrically until it is “trapped” between vessel walls and reaches an ellipsoidal shape.
- The center of mass at this point lies on the medial manifold.



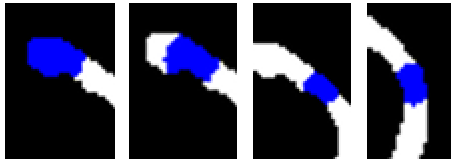
3. PCA-Based Analysis

- Apply Principal Component Analysis (PCA) to the voxel set.
- Detect transition from spherical → elongated shape.
- Compute cross-sectional radii and determine tube direction.



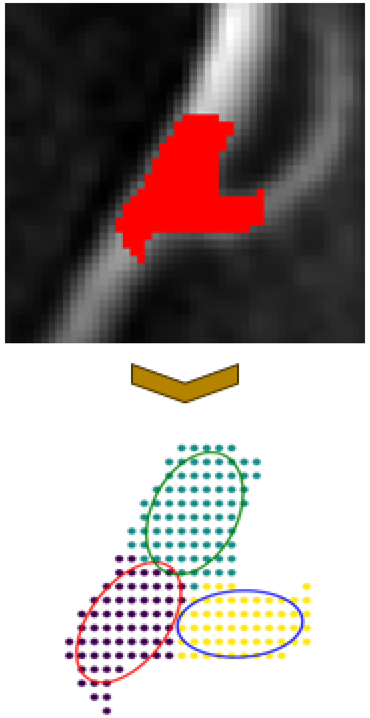
4. Extension Along Tubelike Structure

- Repeat accretion with new seeds along the vessel direction.
- Cover the full vessel length systematically.



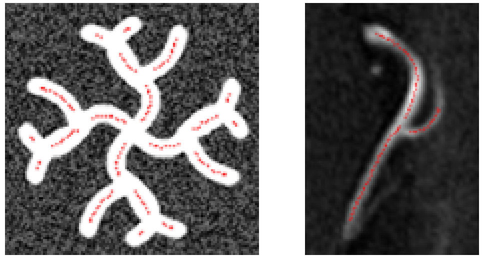
5. Branch Point Detection

- Use Gaussian Mixture Models (GMMs) to detect and parameterize branch points.
- Model each branch as a distinct ellipsoidal population.

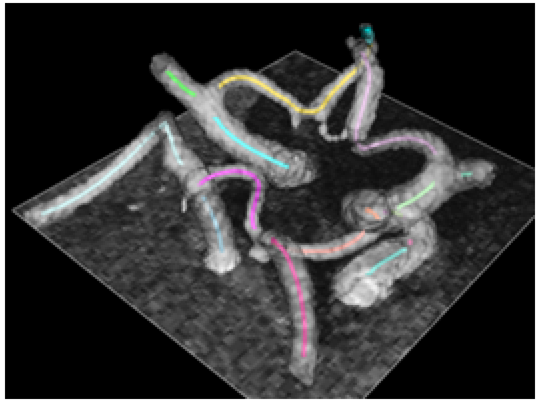


6. Implementation

- Demonstrated in 2D and 3D models.
- Applied to MRI brain vessel data as proof of concept.



2D Medial manifolds shown as a string of points



3D medial manifolds of Circle of Willis arteries, extracted from segmented TOF-MRA scans

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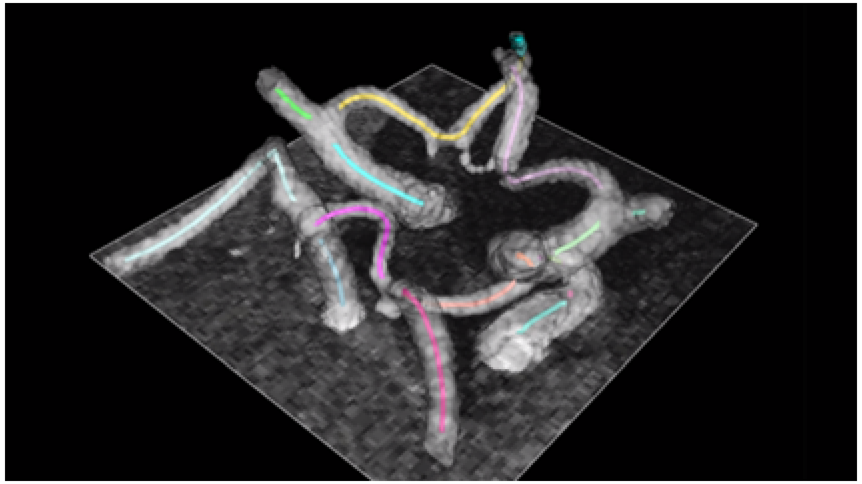
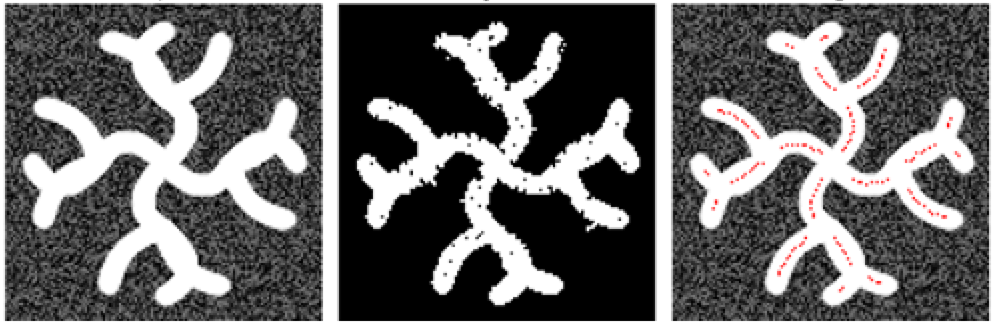
RESULTS



Visualization of the ellipsoidal voxel set (cyan) moving through segmented vasculature in 3D.

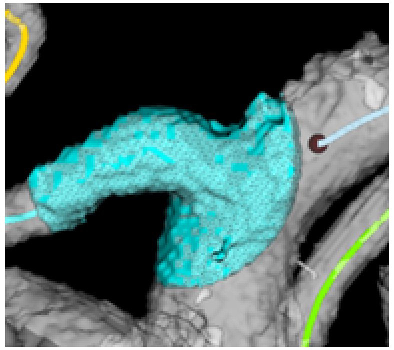


PCA-based ellipsoidal method extracts the medial axis robustly, even with boundary irregularities and noise (left: original vessel, middle: noisy segment, right: medial manifold)

CONCLUSIONS AND FUTURE DIRECTIONS

- Developed a statistically grounded, PCA-based method for medial manifold extraction.
- Demonstrated robustness to noise and irregular boundaries in 2D and 3D vasculature.

- **Limitations**:
  - Current approach not yet generalizable to all tubular structures.
  - Branch point detection is susceptible to errors with different branch sizes
- **Future directions**: broaden applicability and remove remaining heuristic steps to make the method fully statistical.



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