

**Automated Identification and Measurement of Cardiac Anatomy  
via Statistical Analysis of Medial Primitives**

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## ABSTRACT

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### **Automated Identification and Measurement of Cardiac Anatomy via Statistical Analysis of Medial Primitives**

(Under the direction of Stephen M. Pizer, Ph.D.)

Identification and measurement of objects in 3D images can be automatic, rapid and stable, based on local shape properties derived statistically from populations of medial primitives sought throughout the image space. These shape properties are measured at medial locations within the object and include scale, orientation, endness, and medial dimensionality. Medial dimensionality is a local shape property differentiating sphere, cylinder, and slab, with intermediate dimensionality also possible. Endness is a property found at the cap of a cylinder or the edge of a slab. A model of the cardiac left ventricle during systole is constructed as a large dark cylinder with an apical cap at one end, terminated at the other end by a thin bright slab-like mitral valve. Such a model, containing medial shape properties at just a few locations, along with the relative distances and orientations between them, is intuitive and robust and permits automated detection of the left ventricular axis *in vivo* using Real-Time Three Dimensional (RT3D) echocardiography. The statistical nature of these shape properties allows their extraction even in the presence of noise and permits statistical geometric measurements without exact delineation of boundaries, as demonstrated in determining the volume of balloons and of *in vivo* left ventricles in RT3D scans. The inherent high speed of the method is appropriate for real-time clinical use.

## **THESIS**

Fully automated, rapid and robust identification and measurement of cardiac structures in 3D ultrasound images can be achieved by establishing homologies between clusters of medial primitives and the nodes in a medial model of the heart, using statistical analysis of location, scale, orientation, medial dimensionality, and endness.

*"The poet's eye, in a fine frenzy rolling,  
Doth glance from heaven to earth, and earth to heaven,  
And as imagination bodies forth  
The forms of things unknown, the poet's pen  
Turns them to shapes, and gives to airy nothing  
A local habitation and a name."*

King Theseus, from *A Midsummer Night's Dream*,  
by William Shakespeare

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## LIST OF ABBREVIATIONS

2D	Two-Dimensional
3D	Three-Dimensional
AMV	Apex-to-Mitral Valve (axis)
ECG	Electrocardiogram
CO	Cardiac Output
CT	Computerized Tomography
DSR	Dynamic Spatial Reconstructor
GBPT	Gated Blood Pool Tomography
LA	Left Atrium
LV	Left Ventricle, Left Ventricular
m-reps	medial representations
MNM	medial node model
MRI	Magnetic Resonance Imaging
MUGA	Multiple Gated Acquisition
MV	Mitral Valve
RA	Right Atrium
RMS	Root Mean Squared
ROI	Region of Interest
RT3D	Real Time Three-Dimensional
RV	Right Ventricle
SPECT	Single Photon Emission Computerized Tomography
SV	Stroke Volume

## LIST OF SYMBOLS

$\mathbf{x}$ or $\underline{\mathbf{x}}$	vector (in the sense of a location)
$\bar{\mathbf{c}}$	vector (in the sense of an oriented distance)
$\hat{\mathbf{v}}$	normalized vector (orientation)
$\mathbf{C}$	matrix
$a$	scalar
$[a, b]$	range from $a$ to $b$
$F$	operator or function
$O(n^2)$	order $n^2$
$\mathfrak{R}^m$	$m$ - dimensional space of real numbers