Discrete Period Quadrature for Time-Frequency Analysis of Nystagmus Eye Motion

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ABSTRACT

Individuals with nystagmus, a visual impairment characterized by involuntary, repetitive eye movements, perceive an unstable visual field, reducing their quality of life. The presented research contributes to the development of a novel aide for individuals with nystagmus – a mobile application (app), *StabilEyes*, for any smart device – to be used during daily activities. *StabilEyes* will use the front-facing camera to detect the user's unwanted periodic eye motion and translate real-time images captured by the back-facing camera on the screen, effectively stabilizing the visual field of the user. We have developed Discrete Period Quadrature (DPQ), a method for time-frequency analysis, for the purpose of tracking this periodic eye motion. DPQ operates on a discrete signal and produces complex values for a set of frequencies. The resulting magnitude and phase spectra have various features by which the fundamental signal period may be inferred. These features include maximum magnitude at the signal period, local minimum magnitudes at the signal period subharmonics, minimum variance of magnitude at the signal period, and minimum root-mean-squared (RMS) phase-change error at the signal period. We compared the capability of each of the four spectral features identified to infer the period and phase of a sinusoidal signal whose period was changed at various rates or that had various amounts of noise added. Some observations using non-sinusoidal signals (particularly ramp waves) were also made. Preliminary analyses suggest that finding the minimum RMS error between the observed and expected rate of phase-change is the optimal method for tracking a periodic signal with DPQ.

BACKGROUND

In the continuous-time domain, the Fourier Transform operates on a signal over a time window of $(-\infty, \infty)$ to determine the magnitude and phase of all constituent frequencies. The Discrete Fourier Transform (DFT) is the discrete-time equivalent.

Our method, DPQ, is founded upon the same concepts as the DFT and produces magnitude and phase spectra that are comparable to those of the DFT. However, unlike the DFT, DPQ utilizes a window whose size adapts to exactly one discrete period for each frequency considered. This optimizes DPQ for rapid and accurate tracking of a periodic signal.

For our purposes, DPQ must remain both fast and accurate in situations where (1) the fundamental frequency is changing and (2) noise is present.

PURPOSE STATEMENT

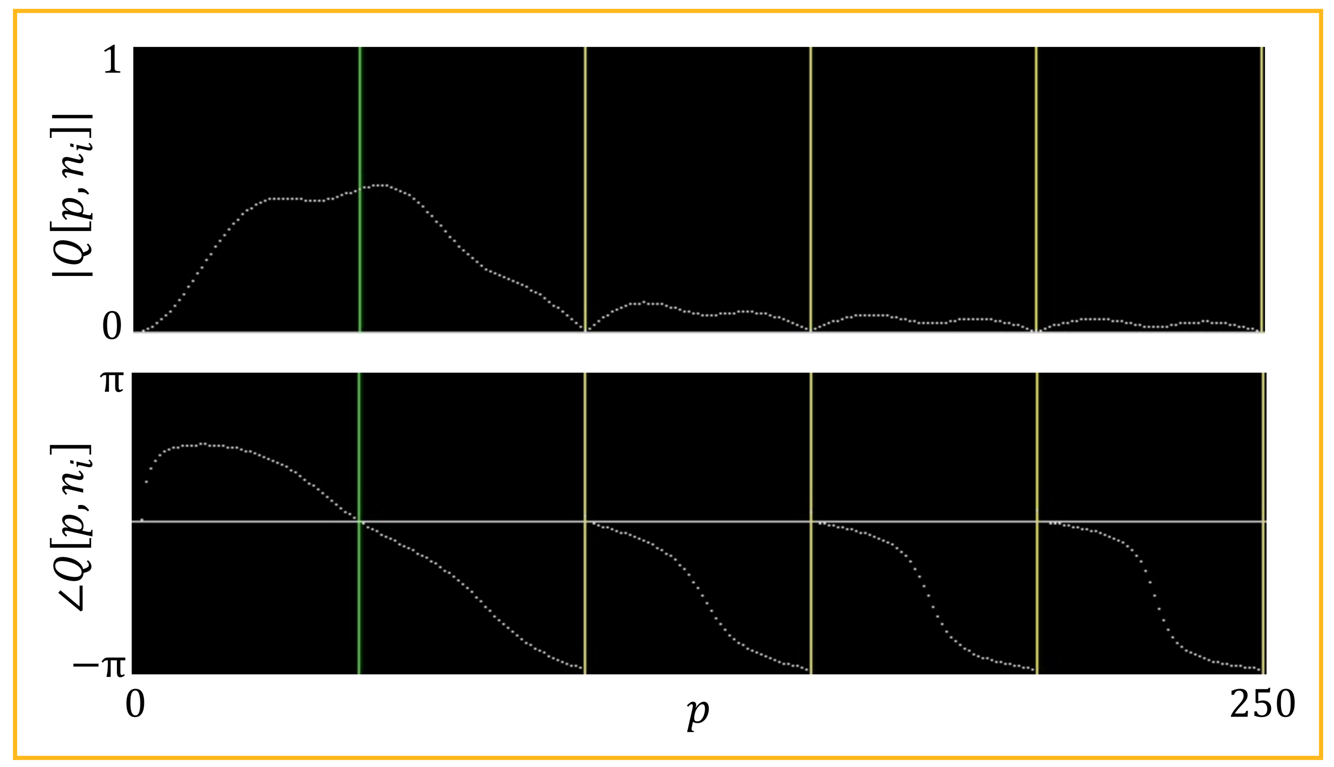
The presented work investigates the use of various spectral features produced by DPQ to infer the frequency and phase of a periodic signal, especially with a change in the signal period or in the presence of noise.

METHODS

DPQ operates on a discrete signal, $s[n]$, yielding complex values for a set of frequencies. Each frequency ω radians/sample has a corresponding integer period p samples, such that $\omega = \frac{2\pi}{p}$. We denote DPQ as $Q[p,n]$ and define it

$$Q[p,n] = \sum_{m=n}^{n-p+1} \frac{s[m]\left(\cos\left(\frac{2\pi(m-n)}{p}\right) + j\sin\left(\frac{2\pi(m-n)}{p}\right)\right)}{p}, 2 \le p \le P$$

where the sum operates on p samples preceding and including the sample at index n. For each p between 2 and a maximum P, this yields the quadrature covariance at each discrete frequency ω as a complex number. One example of the resulting magnitude and $|Q[p,n]|$ phase $\angle Q[p,n]$ spectra is shown below for a signal with a period of 50 samples/cycle.



The green and yellow vertical lines denote the (true) signal period p_s and its subharmonics (integer multiples of p_s), respectively.

We identified the following four spectral features as useful in inferring p_s:

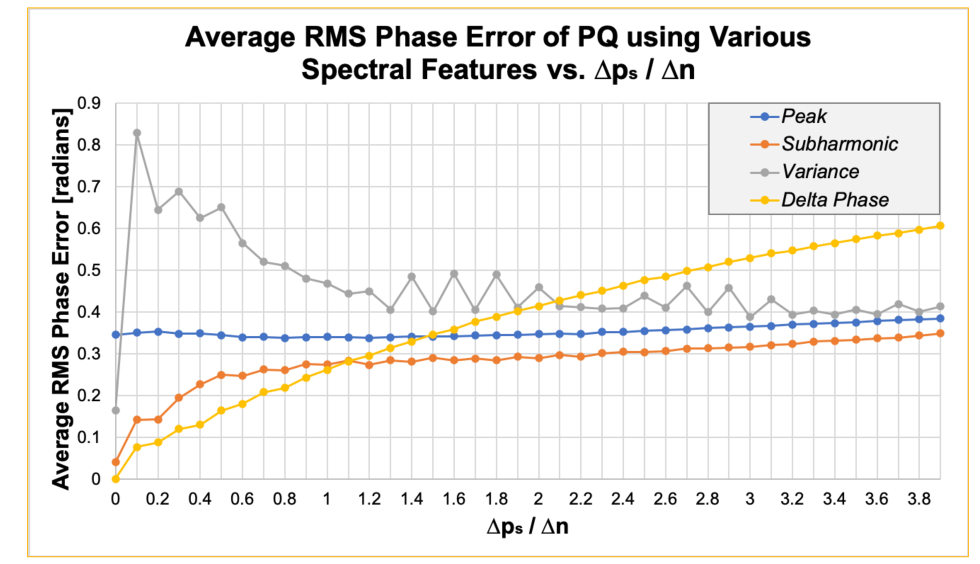
- Peak: $|Q[p,n]|$ has a global maximum at $p \cong p_s$
- Subharmonic: $|Q[p,n]|$ has local minima at $p \cong kp_s$, $k = 2, 3, 4, \ldots$
- Variance: $VAR(|Q[p,n]|)$ has local minima at $p \cong kp_s$, $k = 1, 2, 3, \ldots$
- Delta Phase: RMS error of $[\angle Q[p,n] - \angle Q[p,n-1]] - \frac{2\pi}{p}$ has a global minimum at $p \cong p_s$

We conducted two experiments using a sinusoidal signal with **(1) a change in signal period** and **(2) Gaussian noise**, in which the phase corresponding to each inferred period was compared to the true phase of the signal at each index.

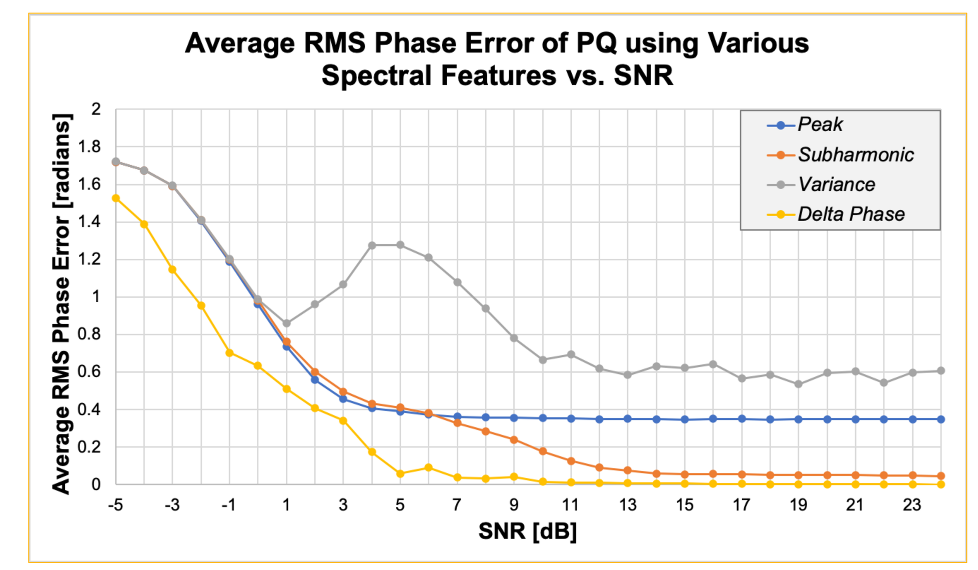
We also observed the real-time performance of each feature using a **non-sinusoidal (ramp) waveform** as the input signal.

RESULTS & DISCUSSION

1. Using the Delta Phase feature resulted in the smallest RMS error between the signal and inferred phases until a rate of change $\frac{\Delta p_s}{\Delta n}$ of about 1



2. Using the Delta Phase feature resulted in the smallest RMS error between the signal and inferred phases for all signal-to-noise ratios.



Observing the performance real-time with a ramp wave signal resulted in the Delta Phase feature inferring period and phase with the most precision and accuracy.

CONCLUSIONS

Inferring p_s using the Delta Phase feature yielded the most consistent and accurate tracking overall. We believe that the nearly linear relationship between average RMS phase error and the rate of signal period change produced using Delta Phase is due to the *memory* of this feature, which will be advantageous in practice. Given an input signal derived from our previously developed eye-tracking method [1], we will employ DPQ to determine the period and use the corresponding phase to control the image translation in our *StabilEyes* app.

ACKNOWLEDGEMENTS

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REFERENCES

1. J. Foust, et al., "StabilEyes - New Assistive Technology for Nystagmus to Produce a Stabile Real-Time Video Image" at BMES 2019 Annual Meeting, Philadelphia, PA, 2019.

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