

1. A point-spread function $h(x,y)$, also known as the impulse response, is defined by the following equation:

$$h(x,y) = \begin{cases} 2 - \sqrt{x^2 + y^2} & , \quad x^2 + y^2 < 4 \\ 0 & , \quad \text{otherwise} \end{cases}$$

- A. What basic geometric shape does the central portion of this impulse response have?
- B. Write an equation (as simple as possible for the same point spread function rotated by 45° clockwise around the origin in the x - y plane. Does $h(x,y)$ exhibit circular symmetry?
- C. Sketch the 1-dimensional function $g(x,0)$, the cross-section of the image $g(x,y)$ along the x -axis, where

$$g(x,y) = f(x,y) * h(x,y)$$

and

$$f(x,y) = \delta(0,0) + \delta(x-2,0)$$

with x plotted along the horizontal axis and intensity plotted on the vertical (label values).

D. What is the FWHM of $h(x,y)$?

2. Two pennies are tossed onto a table, each with equal probability of heads vs. tails, and the total number of heads h determined for each toss. For the corresponding random variable H ,

- A. Sketch the probability mass function or the probability density function for H , whichever is appropriate (indicating which you have drawn), labeling the vertical and horizontal axes.
- B. Sketch the probability distribution function (cumulative distribution function) labeling the vertical and horizontal axes.
- C. What is the mean and standard deviation of H ? (hint, use the equations for mean and standard deviation for a discrete random variable described in http://en.wikipedia.org/wiki/Standard_deviation):

$$\sigma = \sqrt{\sum_{i=1}^N p_i (x_i - \mu)^2}, \quad \text{where } \mu = \sum_{i=1}^N p_i x_i.$$

- 3.** A diagnostic imaging protocol for detecting liver cancer has a 80% sensitivity and a 50% specificity.
- A. Given a prevalence for liver cancer in a particular population of 10%, what is the positive predictive value? (Draw the probability box diagram for + and – disease and test result).
- B. What would the positive predictive value be for a different population with a prevalence for liver cancer of 50%? (Again, draw the probability box diagram)
- 4.** Explain how two point spread functions with the same FWHM can have different modulation transfer functions.