University of Pittsburgh
Department of Bioengineering

BIOE 1330
Biomedical Imaging
Homework #1 Solns

1. Signals are continuous or discrete functions that occur in the time or spatial domain. Systems are "black boxes" that modify signals. How do signals and systems apply to medical imaging?

   If an image is considered to be a two-dimensional discrete signal, systems theory allows us design desirable modifications for the image. This approach is the theoretical basis of image processing.

2. If an image is a 2-D discrete signal and all of a system's characteristics can be defined by the system's impulse response, then convolution can describe passing the image through the system. What is the advantage of using the Fourier transform and performing analysis in the Frequency domain?

   The mathematical (processing time) overhead for signal analysis and modification can be MUCH less when performed in frequency domain than when applying convolution. This is true, even given that two Fourier transform calculations are required to get there and back again.

3. Why is separability a desirable characteristic of an image operation?

   Not only is the math easier, but it allows images to be processed in a single dimension.

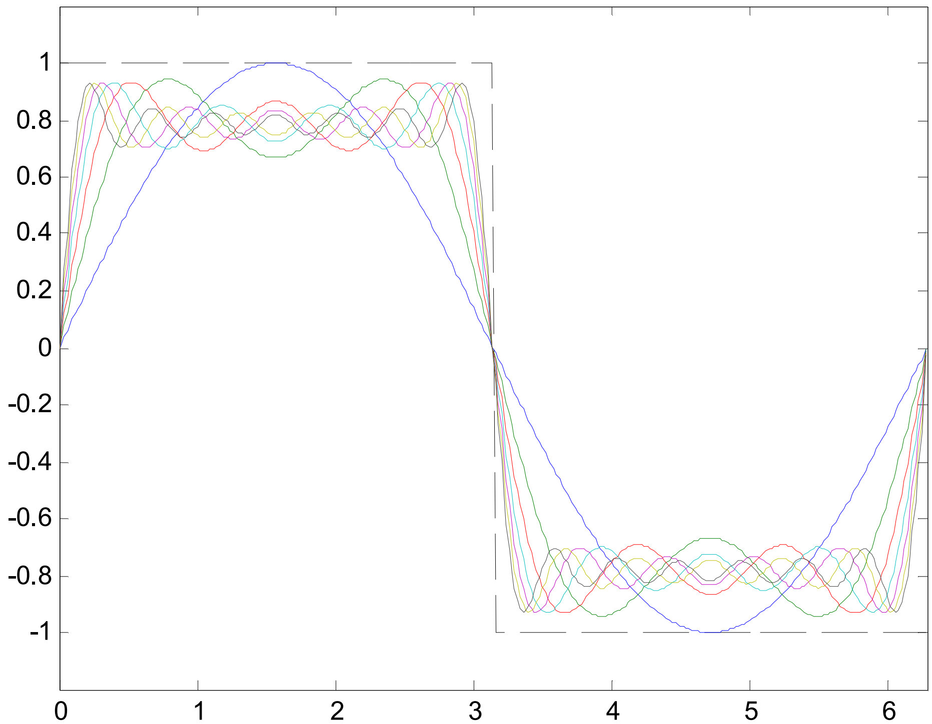
4. Consider $\delta(x)$
   a. What is its value for $x \neq 0$?
   b. What is its value for $x = 0$?
   c. What is its frequency content?
   d. What is the area of this function?

   a: 0, b: infinity, c: all frequencies equally, d: 1 by definition

5. Convolution in the time or space domain results in multiplication in the frequency domain. Convolving a linear shift- invariant system with the delta function produces the impulse response or point spread function of the system. This is the same as multiplying the frequency response by 1 for all frequencies. What is the use of determining the impulse response?

   Any input signal can be convolved with the impulse response to determine the resulting output when that signal goes through the system.

6. There is a MATLAB example with code at http://www.mathworks.com/products/matlab/demos.html?file=/products/demos/shipping/matlab/xfourier.html.
   Provide the matlab graphical output for the first 13 harmonics plotted against the square wave that they approximate. Explain what happens as more harmonics are added.



   As more harmonics are included the signal becomes a better approximation of the square wave. An infinite number of harmonics would be needed to reconstruct the square wave.

7. Determine if the following signals are separable.
   a. $\delta(x,y) = \delta(x) \cdot \delta(y)$, separable

   b. $f(x,y) = e^{2\pi j(2x+3y)}$, $f(x,y) = e^{2\pi j(2x+3y)} = e^{2\pi j 2x} \cdot e^{2\pi j 3y}$, separable

   c. $f(x,y) = sin\big(2\pi(2x+3y)\big)$, not separable

8. Determine whether the following signals are periodic. If they are, find the fundamental period
   a. $x(t) = e^{j5t}$
      $e^{j5t} = e^{j5(t+T)} = e^{j5t}e^{j5T}$
      In order to be periodic,
      $e^{j5T} = 1$

$5T = 2k\pi$
$T = \frac{2\pi}{5}$, Periodic

b. comb(x, y)
comb $(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{8} \delta(x - m, y - n)$

$$(x + M, y + N) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{8} \delta(x - m + M, y - n + N)$$

Let p=m-M, q=n-M
$(x + M, y + N) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{8} \delta(x - p, y - q)$ = comb(x, y),
Periodic, $Tx = 1, Ty = 1$

c. $g[m, n] = \sin m \cos \pi n$ Note: $g[m, n]$ is a discrete signal
$\sin m = \sin(m + T_m)$
$T_m = 2k\pi$
$T_m = 2\pi$, smallest period, but since g is a discrete signal, the period must be an interger, so not periodic in m
$\cos \pi n = \cos\big(\pi(n + T_n)\big)$
$\pi T_n = 2k\pi$
$T_n = 2$, periodic in n, with a period of 2.

9. Evaluate the following:
   a. $f(x)\delta(x)$, where $f(x) = e^x$
   $f(x)\delta(x) = e^x\delta(x) = e^0\delta(x) = 1\delta(x) = \delta(x)$

   b. $f(x, y) * \delta(x, y + 4)$, where $f(x, y) = x + \sqrt{y}$

$$f(x, y) * \delta(x, y + 4) = \iint_{-\infty}^{\infty} f(\xi, \eta)\, \delta(x - \xi, y - \eta + 4) d\xi d\eta$$

$$= \iint_{-\infty}^{\infty} f(x, y + 4)\, \delta(x - \xi, y - \eta + 4) d\xi d\eta$$

$$= f(x, y + 4) \iint_{-\infty}^{\infty} \delta(x - \xi, y - \eta + 4)\, d\xi d\eta = f(x, y + 4)$$

$$= x + \sqrt{y + 4}$$