Homework 1 Solutions:

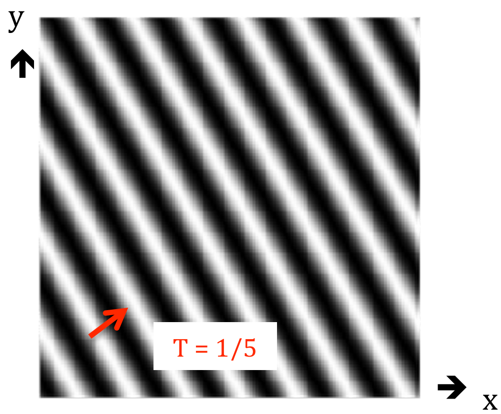
Problem 1 A)

$F(u = 4, v = 3) = (1 + j)\ \delta\ (u-4, v-3) = (1 + j)\ \delta\ (0, 0)$

Therefore, $F(u, v) = ((1 + j)\ \delta\ (u-4, v-3)) + ((1 - j)\ \delta\ (u+4, v+3))$

And $F(u+4 = 0, v+3 = 0) = (1 - j)\ \delta\ (u+4, v+3) =$ **$(1 - j)\ \delta\ (0, 0)$**

Problem 1 B)



The period in the direction of change (which is approximately 37 degrees from the x-axis) is 1/5. Also note the sinusoid consists of both a cosine and sine components and is thus shifted by 1/8 of a period from the origin.

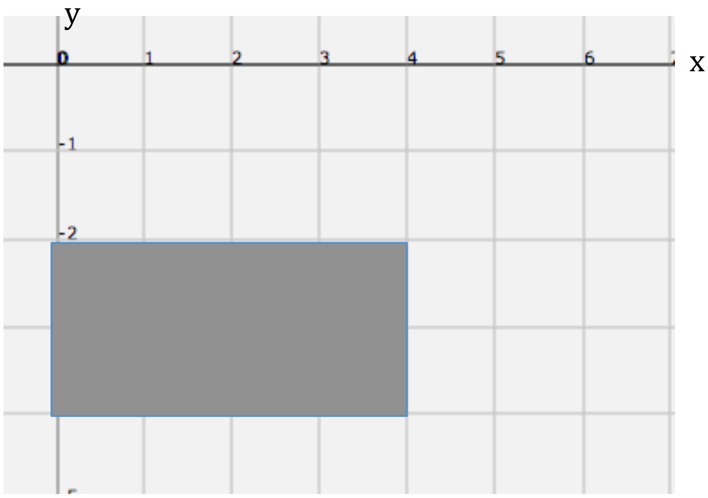
Problem 1 C)

Amplitude of a sinusoid = magnitude of Fourier transform = sqrt(Re(F(u,v))^2 + Im(F(u,v))^2)

Amplitude of sinusoid in this problem = sqrt((1+1)^2 + (1+1)^2) = sqrt(8) = 2.83

Since the sinusoid is centered along the xy plane, the maximum intensity is equal to the amplitude (**2.83**)

Problem 2 A)

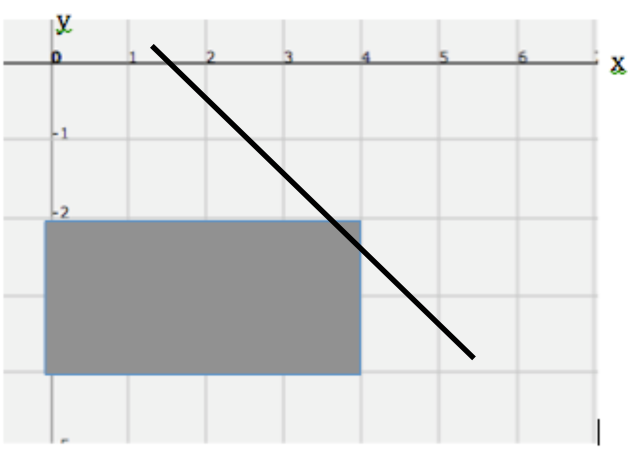


Problem 2 B)

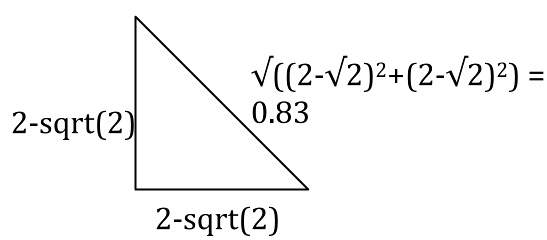
Intensity of shaded region = π*Area of rectangle = π*4*2 = **<u>8π</u>**

Problem 2 C)

l = xcosθ + ysinθ where l = 1, θ=π/4; therefore, y = -x + sqrt(2)



The value of sample = length of the intersection * π = **<u>0.83 π</u>**



Problem 3 A)

spacing in frequency domain = 1/spacing in time domain

$\Delta u = 1/\Delta x =$ **$\underline{1/3}$**

$\Delta v = 1/\Delta y =$ **$\underline{1/4}$**

Problem 3 B)

Aliasing will occur if the spacing in the frequency domain is less than twice the max frequency in corresponding dimension (i.e. $\Delta f \leq 2^* f_{max}$). Therefore, aliasing will occur above the following frequencies:

In u dimension: **$\underline{f_u \geq \Delta u/2 = 1/6}$**

In v dimension: **$\underline{f_v \geq \Delta v/2 = 1/8}$**

Problem 3 C)

To avoid aliasing, low pass filter the signal in analog domain before sampling at frequencies less than of equal to f_u and f_v found in problem 3B in the respective directions.

4 A) A signal is separable if: $f(x,y) = f_1(x) f_2(y)$

$h(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$ ← 2D Gaussian (where $x_0 = y_0 = 0$ & $\sigma_x = \sigma_y = \sigma$)

$h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$ & $h(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-y^2/2\sigma^2}$ ← 1D Gaussian (Given $x_0, y_0, \sigma_x, \sigma_y$ above)

$h(x)h(y) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^2 e^{-x^2/2\sigma^2} e^{-y^2/2\sigma^2} = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}} = h(x,y)$

∴ $h(x,y)$ is separable

B) $\theta = \tan^{-1}(y/x)$
$r^2 = x^2 + y^2$ ∴ $h(r,\theta) = \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$ ← $h(x,y)$ in radial coordinates

∴ for any given radius r, $h(x,y)$ doesn't change for all values of θ since $h(r,\theta)$, and therefore $h(x,y)$, is independent of θ.

∴ $h(x,y)$ is rotationally invariant.

c) As $\sigma \to 0$, $h(x,y) \to \delta(x,y)$ ∴ $H(u,v) = 1$

As $\sigma \to \infty$, $h(x,y) \to 1$ ∴ $H(u,v) = \delta(x,y)$

Note: Visualize the transformation of a guassian curve as $\sigma \to 0$ & $\sigma \to \infty$