1. An 2D image $f(x,y)$ consists of a single real sinusoidal component. The Fourier transform $F(u,v)$ of the image at $u = 4$ and $v = 3$ is $(1+j)\,\delta(u\text{-}4,v\text{-}3)$.

A. What is the value of $F(u+4,v+3)$?

B. Using pencil lines to depict the "crests" of the sinusoidal component (it's maximum intensity), sketch the image $f(x,y)$, carefully labeling the axes including units and paying close attention to the correct frequency and phase of the sinusoidal component.

C. What is the maximum intensity in the image?

2. An image consists of the function

$$f(x,y) = \pi \ \mathrm{rect}\left(\frac{x-2}{4}, \frac{y+3}{2}\right)$$

A. Sketch the image using shading to show the portion of the image that is non-zero.

B. What is the intensity of the shaded region?

C. A line impulse function

$$L(\ell,\theta);\ \ \ell=1,\ \theta=\pi/4$$

is used to sample the image

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\ \delta_\ell(x,y)\,dxdy\ .$$

Include the line impulse in the sketch of the image in (A). What is the value of the sample for this particular image?

3. An impulse train

$$\delta_s(x,y;\Delta x,\Delta y);\ \ \Delta x = 3, \Delta y = 4$$

is used to sample an image.

A. Given that the Fourier transform of an impulse train is another impulse train, what is the spacing in each of the u and v dimensions of the fundamental period of the spectrum of the image, irrespective of the particular image sampled?

B. Above what frequency in each of the *u* and *v* dimensions will aliasing occur?

C. What must be done to avoid aliasing (be specific in terms of whether this must occur before or after sampling)?

4. Prove that the Gaussian

$$h(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

is

A. separable

B. rotationally invariant.

C. Ignoring constant coefficients, what is $H(u,v)$ as $\sigma \to 0$, and as $\sigma \to \infty$?