

Homework #1

BioE 1330 – Spring 2011

1. A single sinusoidal component of a 2D image appears as waves 6 pixels apart, whose direction (in which they would be headed if they were moving across the ocean) is 30 degrees East of North. The crest of one of the waves includes the origin of the x - y plane. Draw a picture of this Fourier component similar to the components shown in Fig 2.5 in Prince's textbook. Based on the equations 2.26 and 2.27 in the book, write an equation using complex exponentials (no trig functions) defining the image you have drawn in units of pixels. Do not simply use u_0 and v_0 but replace with actual values of frequency. In which direction, x or y , is the frequency greater?

2. The intensity f of a particular continuous 2D image is defined in physical space in units of millimeters as

$$f(x, y) = x^2 - 2y$$

Referring to equation 2.14 in the book for the sampling function, what value would be returned by the expression

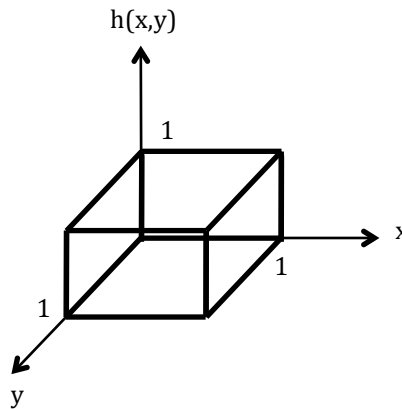
$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \delta(x - m\Delta x, y - n\Delta y) dx dy$$

for the pixel at (3, 4) in pixel coordinates (integers) if the inter-pixel distance in physical space is 2mm? Show your work.

3. Concerning the 1D $\delta(\mathbf{x})$ function:

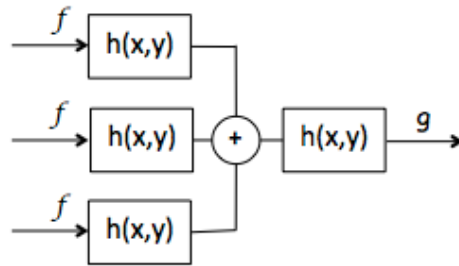
- What is its value with x not equal to 0?
- What is its value at $x=0$? (digital and analog domains)
- What is its frequency content?
- The 1D rect function has a width of 1; i.e. has a non-zero value between $+1/2$ and $-1/2$. What is the width of the delta function?
- What is the area of the delta function?

4. Below is a sketch of an impulse response:



- What is an impulse response?
- Why is this system separable?

- c) Is this system circularly symmetric? Explain?
- d) What type of functions does it separate into (specify what transformation has been applied to the functions)? Write $h(x,y)$ in terms of these functions and delta functions representing the transformations.
- e) What is a transfer function?
- f) Write the equation for the transfer function of this system.
- g) What is the impulse response in terms of $h(x,y)$ of the below system where f and g are input and output signals respectively. For extra credit also sketch the impulse response (a discretized version would also be okay as long as the differences are explained and all work is shown). (Note: all $h(x,y)$ are identical in the figure below and correspond to the above impulse response figure):



5. What affect does aliasing have on an image? How do you avoid aliasing? Explain and be specific.