BioE 1310 – Homework 6

1. Building the Fourier Series for Square and Triangle Waves

Any periodic waveform can be broken down into a series of sinusoids at discrete "harmonic" frequencies $n\omega_0$, where ω_0 is the fundamental frequency in radians per second, and $n = 0,1,2,3....,\infty$ is the harmonic number. One way to describe this mathematically is to build the so-called "Fourier series" of sinusoids, from $\sin$ and $\cos$ functions at each harmonic frequency,

$$V(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right). \qquad (1)$$

The coefficients a_n and b_n thus completely determine the shape of any periodic signal. For example, for a simple sine wave, $b_1 = 1$ with all other coefficient equal to 0. Refer to Section 2.35 in Scherz for a description of the Fourier series and derivations of the series for square and triangle waves. The gist is that one can build a square wave from $\sin$ waves at all of the odd harmonics up to $n = \infty$ with $b_n \propto 1/n$. Such values are shown in the table to the right to 3-place accuracy for the first 13 harmonics. A wonderful interactive webpage for exploring the Fourier Series is available at

http://www.phy.ntnu.edu.tw/java/sound/sound.html

Enter values from the table to the right into the webpage to build a square wave (Caution: use the "set" button rather than the carriage return to enter values). Why is the resulting signal close to, but not exactly, a perfect square wave? **(1)** To build a triangle wave, one uses $\cos$ waves at all of the odd harmonics up to $n = \infty$ with $a_n \propto 1/n^2$. Compute values to 3-place accuracy for the first 15 harmonics of the triangle wave and enter them into the webpage. Given that the derivative of a triangle wave is a square wave, explain the relationship of $a_n \propto 1/n^2$ (triangle wave) to $b_n \propto 1/n$ (square wave). **(2)** Explain the difference you heard in audio tone between the sin, the triangle, and the square waves, in terms of the Fourier series of the three waveforms. **(3)**

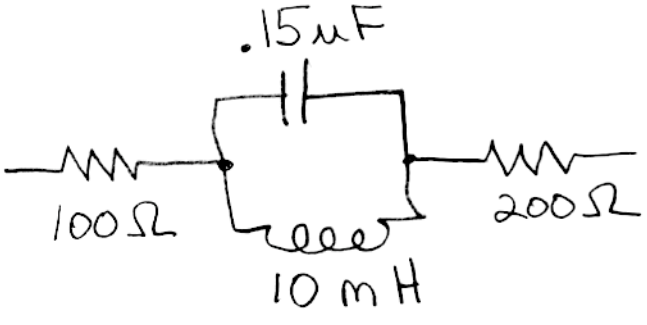
Square Wave

n	a_n	b_n
0	0	0
1	0	1.000
2	0	0
3	0	0.333
4	0	0
5	0	0.200
6	0	0
7	0	0.143
8	0	0
9	0	0.111
10	0	0
11	0	0.091
12	0	0
13	0	0.077
14	0	0
15	0	0.067

④ How many dB does a system change a signal at 5 KHz if the amplitude of a 5 KHz sinusoid changes from 20V to 1mV?

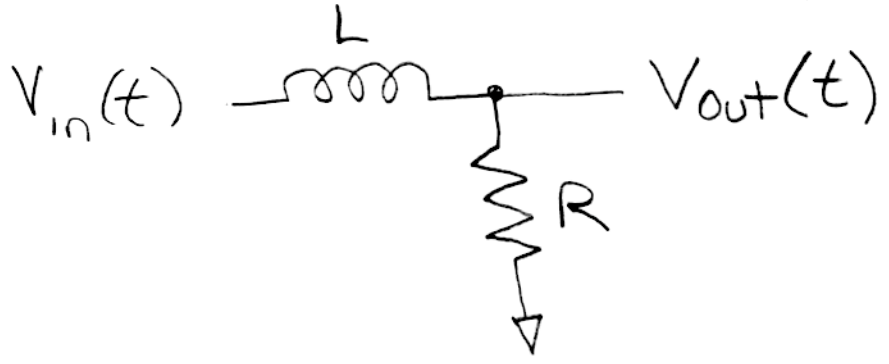
⑤ A sinusoidal pressure wave represents 5 W of power. How would this be expressed in dB relative to a 1nW sinusoidal pressure wave at the same frequency?

⑥ Compute an expression for the total impedace $\mathbb{Z}$ of the following circuit.



What are the maximum and minimum values for $|\mathbb{Z}|$ and at what frequencies do they occur? (Hint: there may be more than one maximum or minimum.)

(7) Consider the following circuit



write an equation for $H(\omega)$, the transfer function. (This is just a divider with impedances instead of resistances)

(8) At what frequency (radians/sec) does $|Z_L| = R$ if $L = 10mH$, $R = 100\Omega$

(9) Is this a high pass or low pass filter?

(10) Given the values in (8), sketch a plot of $|H(\omega)|$ in dB and $\angle H(\omega)$ in radians vs $\log \omega$. (Bode plot)