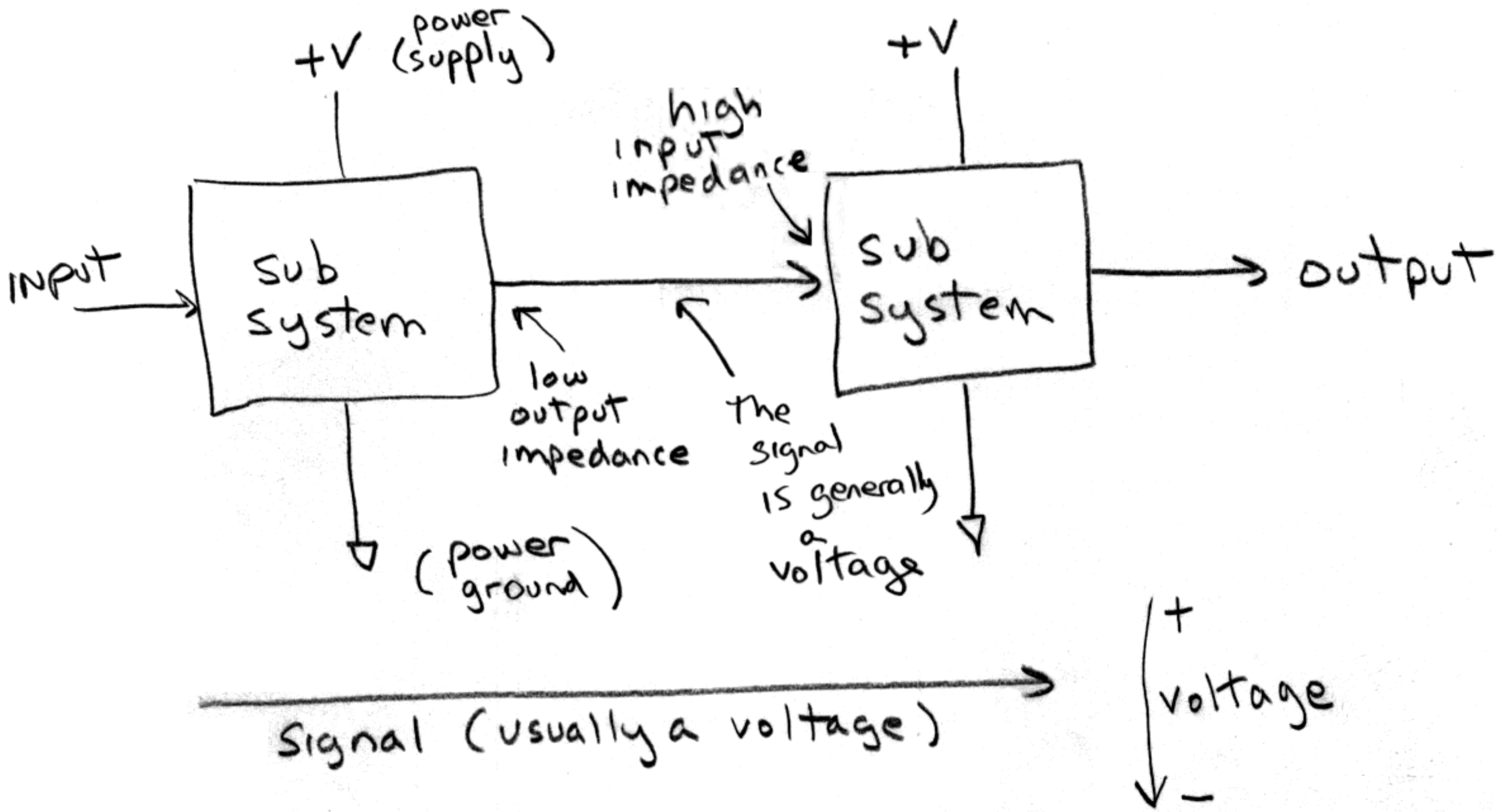
Section 4: Operational Amplifiers

- Op Amps
- Integrated circuits
- Simpler to understand than transistors
- Get back to linear systems, but now with gain
- Come in various forms
  - Comparators
  - Full Op Amps
- Form the building blocks for larger systems
  - Differential Amplifiers
  - H-Bridge Amplifiers

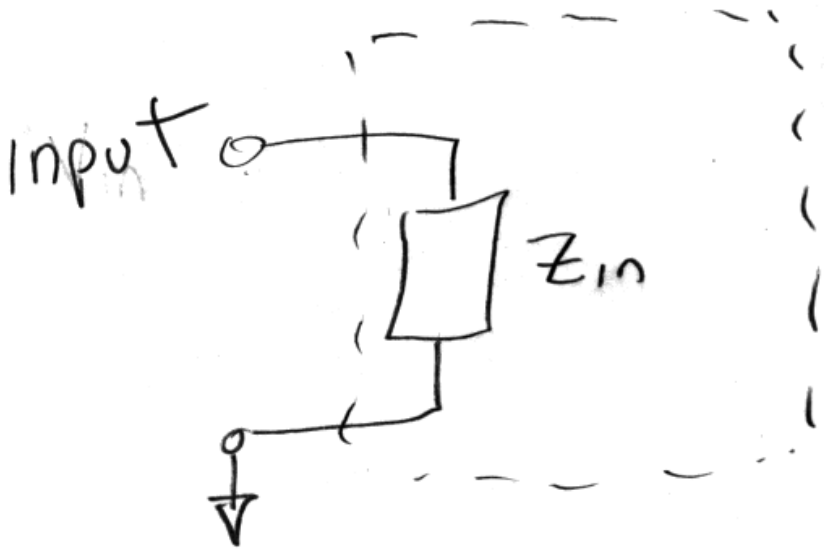
Operational Amplifiers ("Op Amp")

- Integrated Circuit (IC) – complex system on a chip with simple behavior.
- We can basically ignore what goes on inside if we understand that behavior and make some assumptions.
- Since the early 1970's they have dominated analog circuit design.

Systems and Schematics



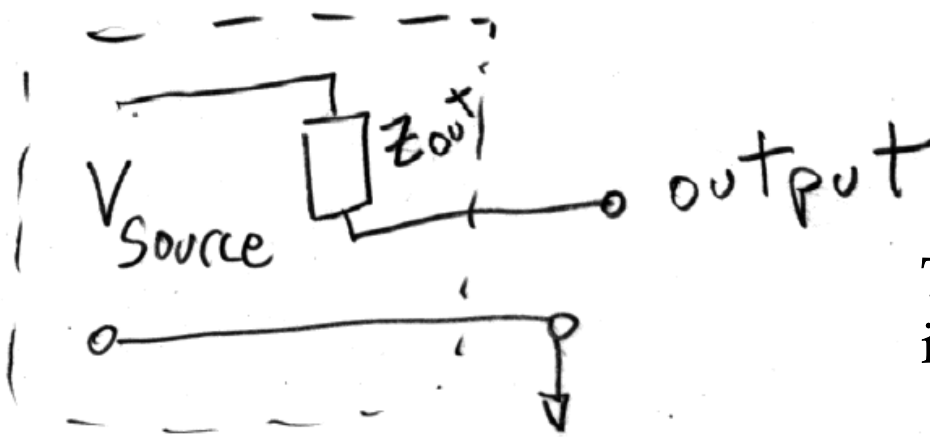
Input and Output Impedance for each subsystem should ideally be infinite for input and zero for output.



Ideally

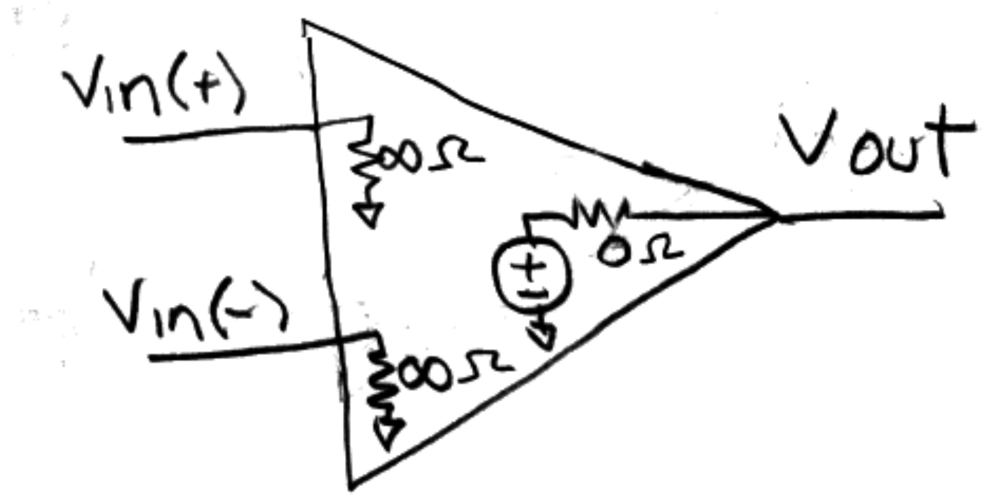
$Z_{in} = \infty$

$Z_{out} = 0$



Thevenin equivalent
is handy here

Operational Amplifier ("Op Amp")



$$V_{\text{out}}=A(V_{\text{in}+}-V_{\text{in}-}),\ \ A\rightarrow\infty$$

Properties of Ideal Op Amp

1. Infinite Input Impedance
2. Zero Output Impedance
3. Infinite Gain

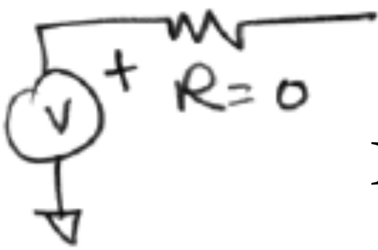
Properties of Ideal Op Amp (continued)

1. Infinite Input Impedance

Reads input voltage without changing it by drawing current.

2. Zero Output Impedance

Can provide infinite output current without effecting voltage.



$$V_{\mathrm{OUT}} = V - I\underset{0}{R} = V$$

Like a battery with zero internal resistance, can drive any input without being affected.

3. Infinite Gain

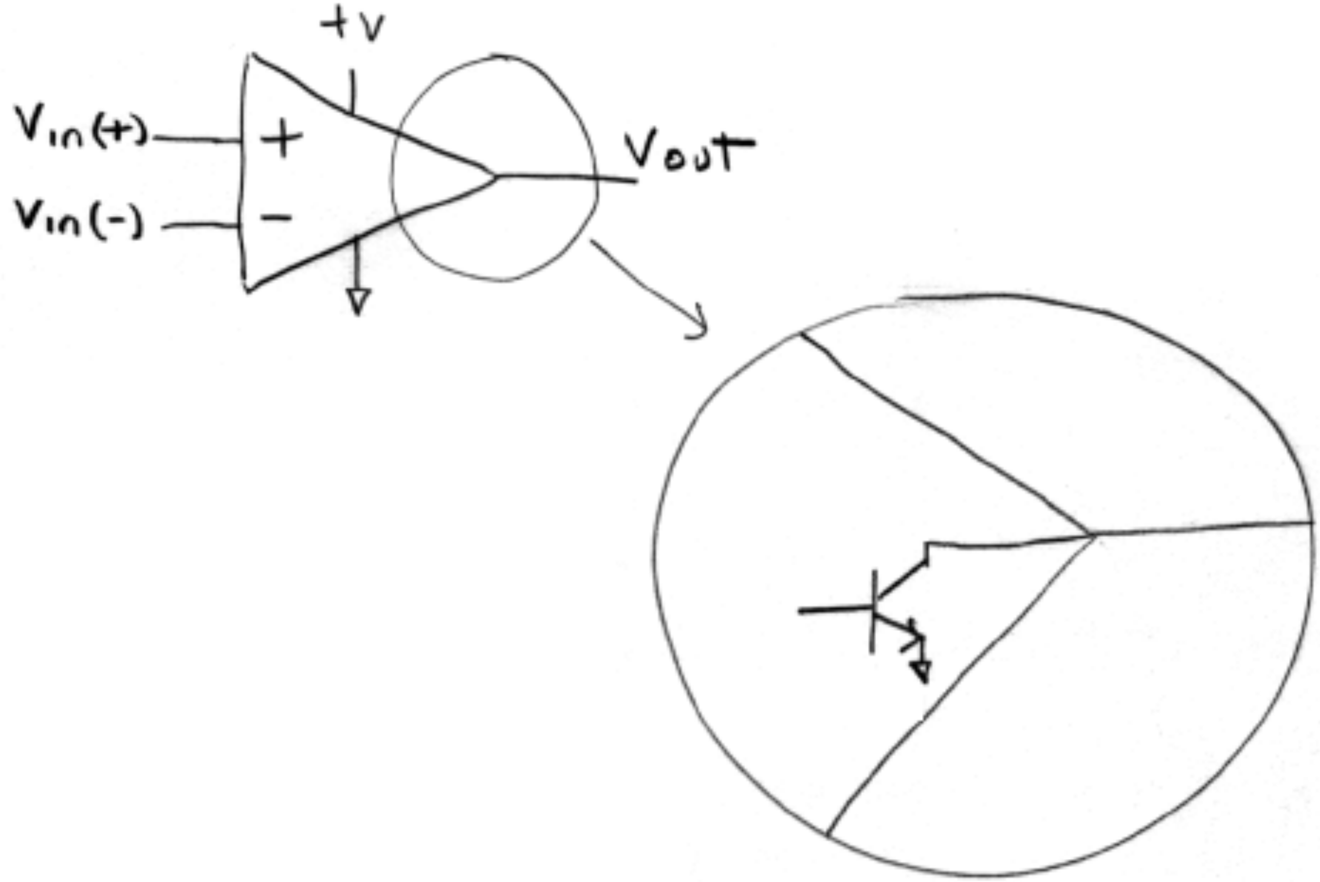
Equal for both inputs, so can have single variable A.

$$V_{\mathrm{out}} = A(V_{\mathrm{in}+} - V_{\mathrm{in}-}), \quad A \to \infty$$

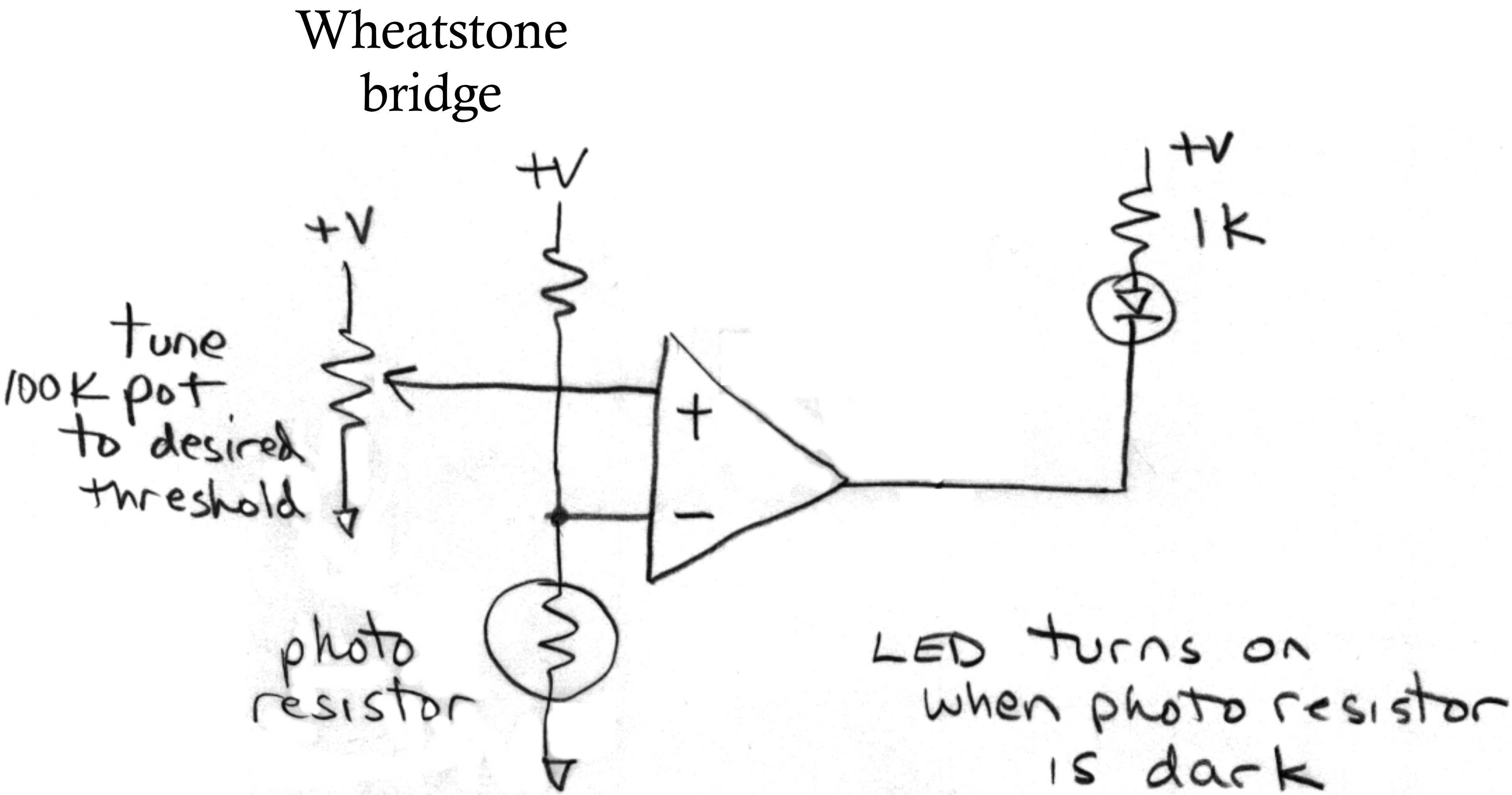
Comparator

- Simple Op Amp for comparing voltages
- Single-Sided Power Supply (just + and ground)
- *Open Collector* output:

  can only *sink* (pull) current, not *source* (push) it.



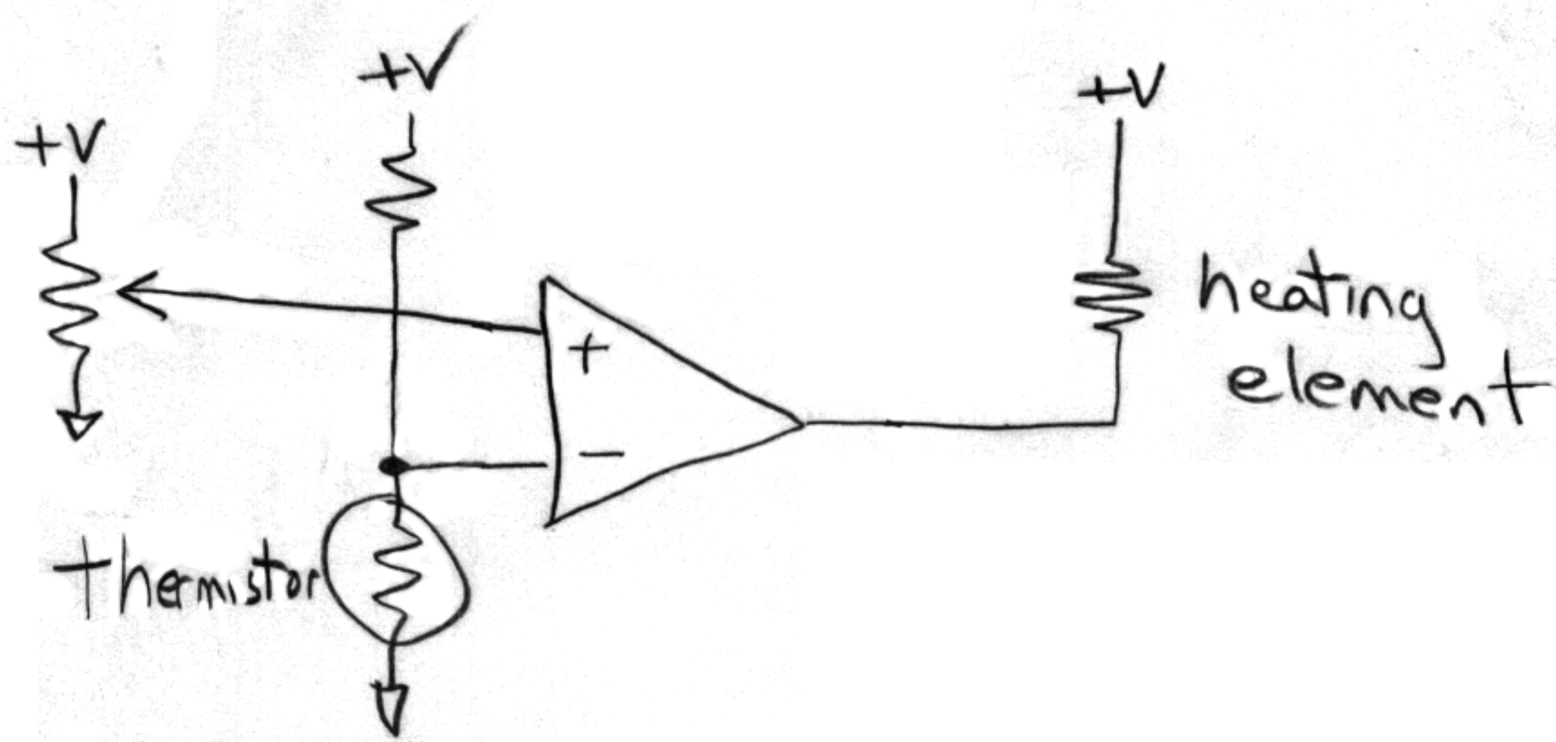
Comparator light-sensing circuit



Basically, comparators have a digital output.

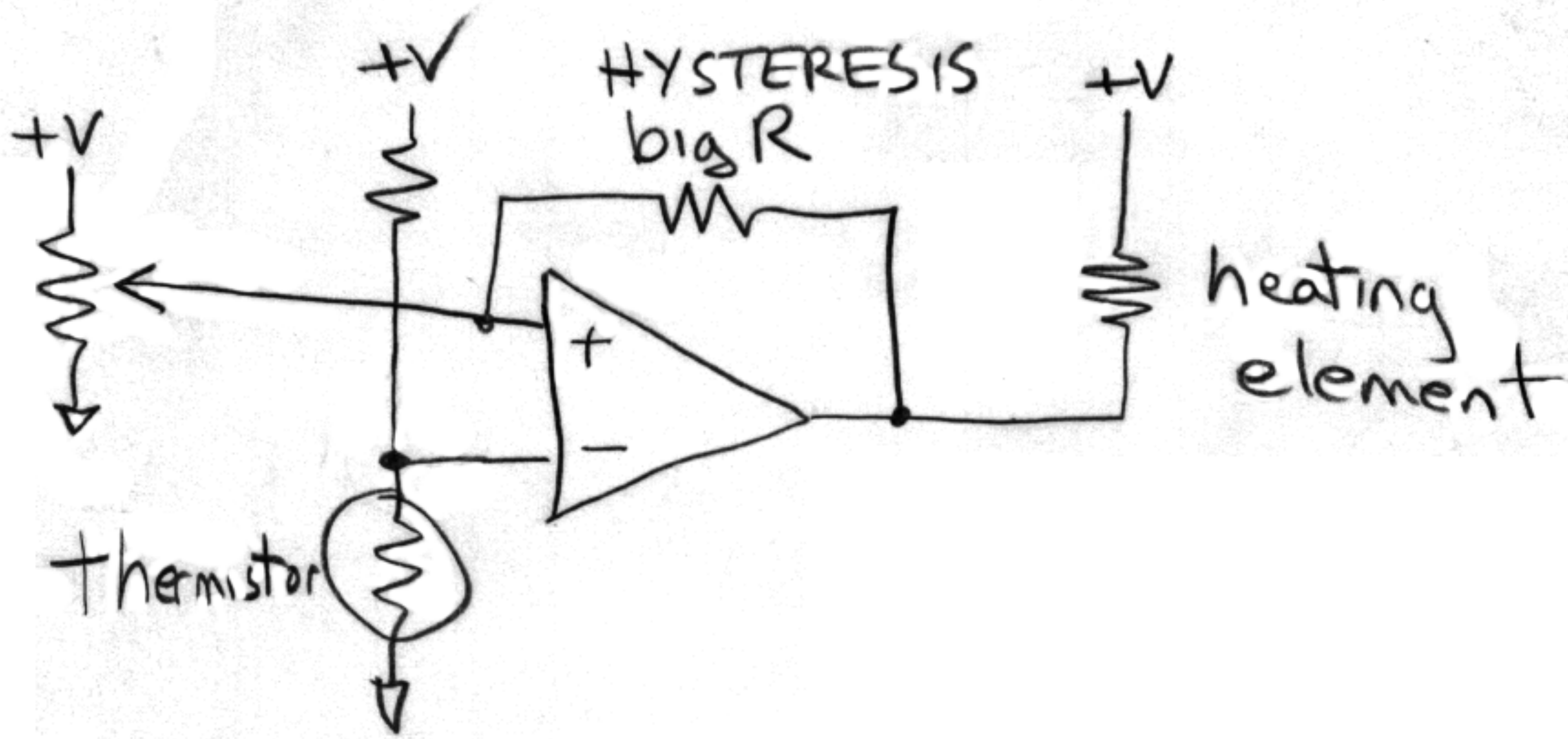
Comparator thermoregulation circuit

Example of Negative Feedback

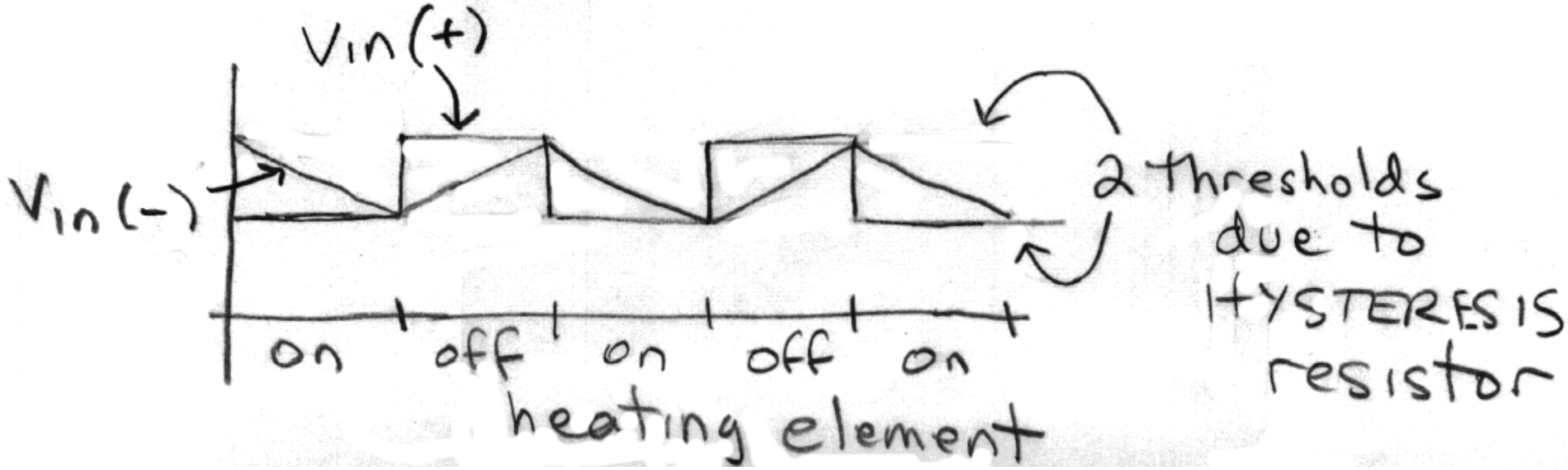


assuming neg coeff $\left(\frac{\Delta\Omega}{\Delta °C}\right)$ thermistor

$\uparrow$ temperature $\Rightarrow \downarrow \Omega \Rightarrow \downarrow V_{in(-)} \Rightarrow$ turn off heater



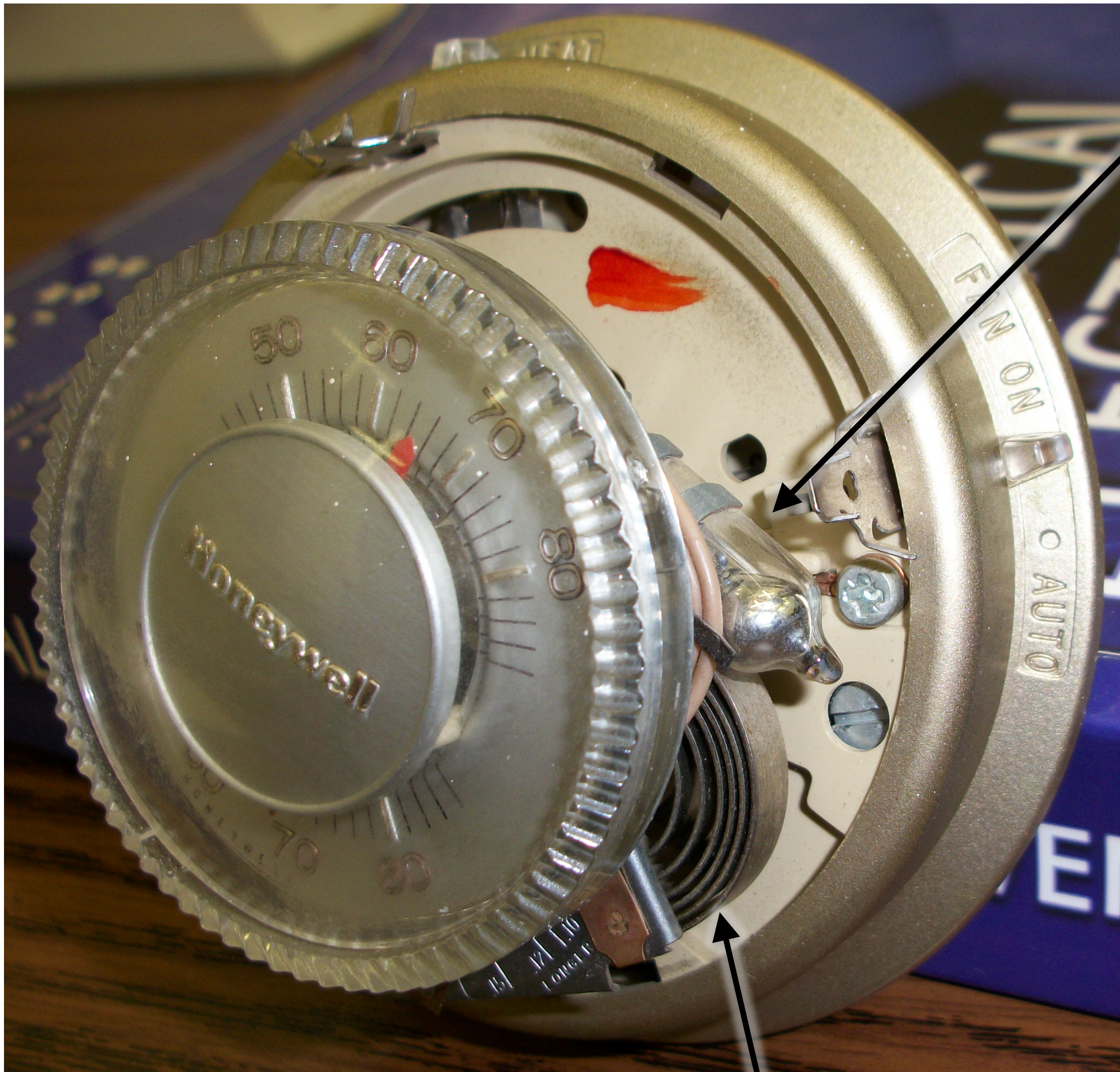
To prevent "chatter" change the set point by adding just a little *positive* feedback to the *negative* feedback system.

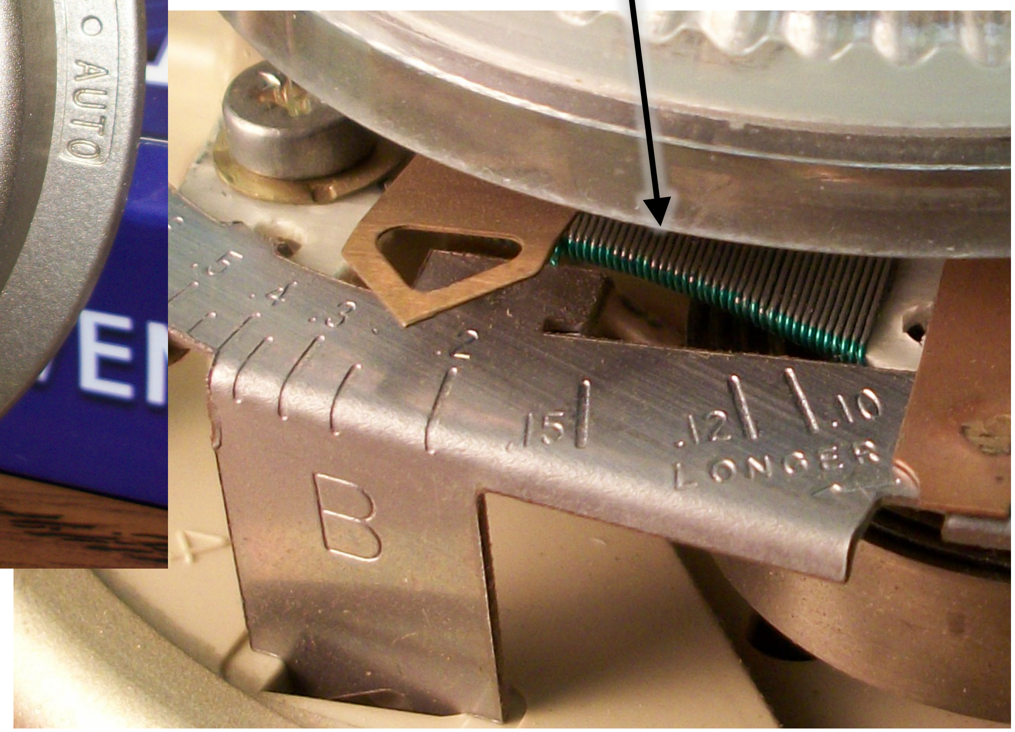


Mechanical Thermostat

Mercury switch tips over to activate furnace

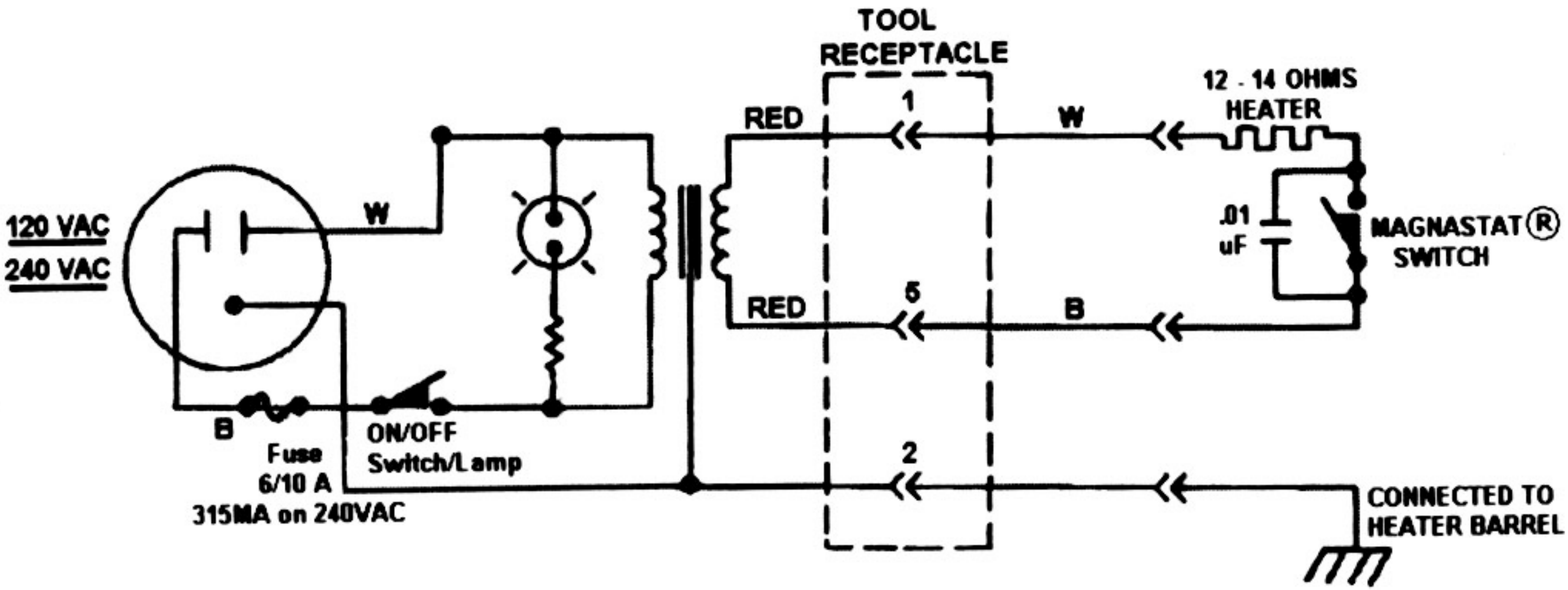
Small local heating coil provides hysteresis: turns on ~~with~~ when furnace is off and ~~raises~~ lowers the “~~off~~” “on” set-point (room must be colder).





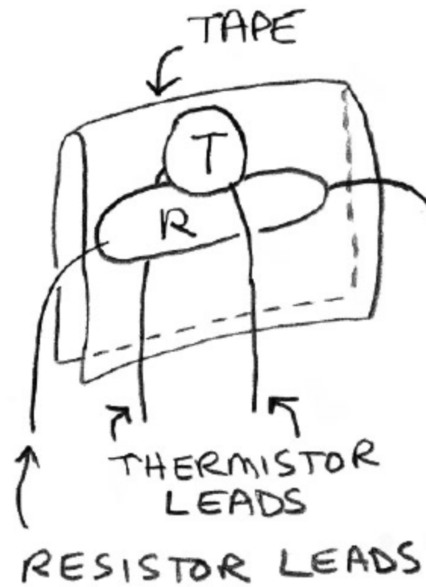
Bilayer metal bends with temperature.

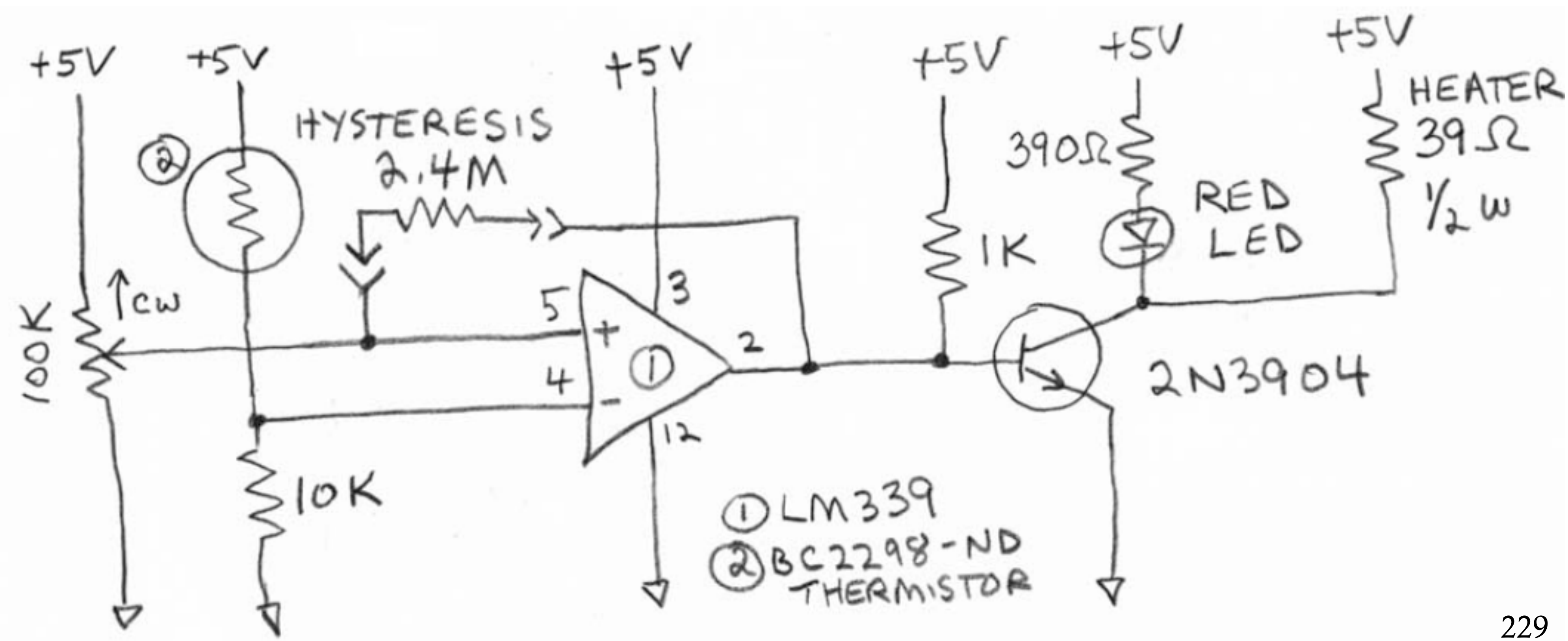
Weller Soldering Iron Station



- Thermoregulation – magnet heats up, loses its magnetism, and releases (opens) switch
- Natural hysteresis in time to heat magnet.

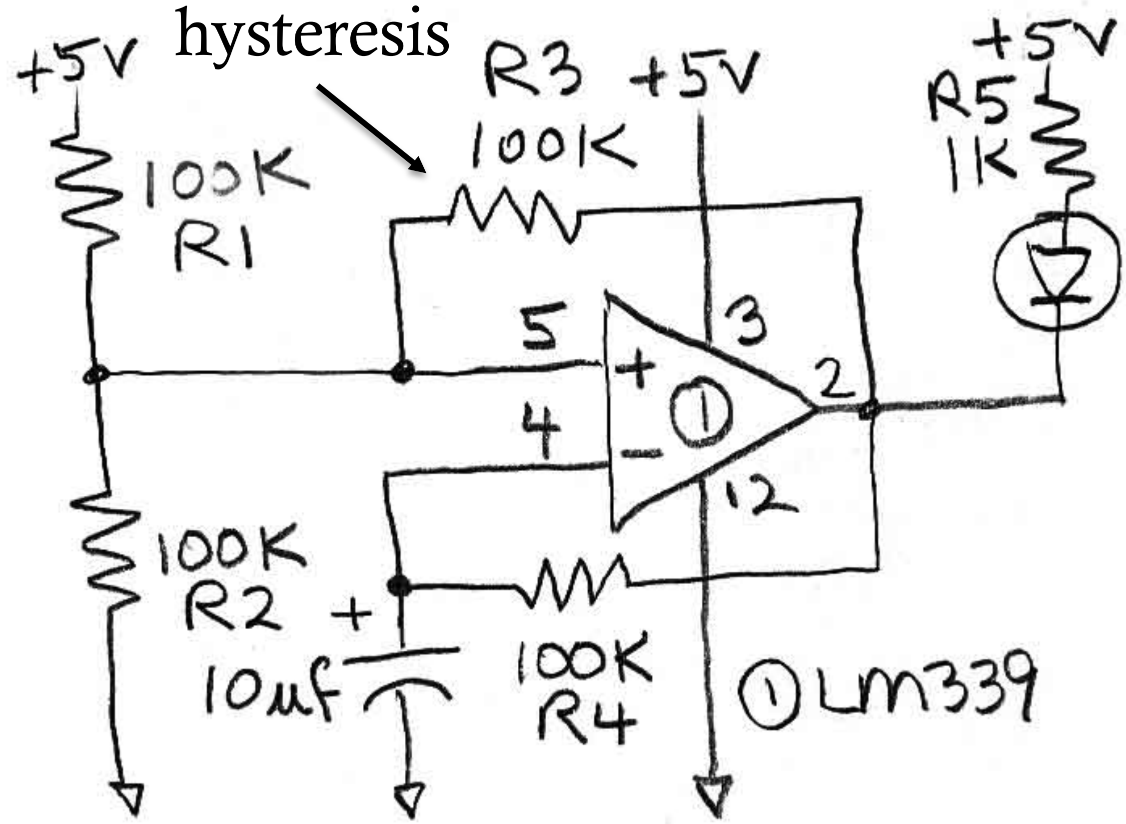
Heater with transistor for current gain

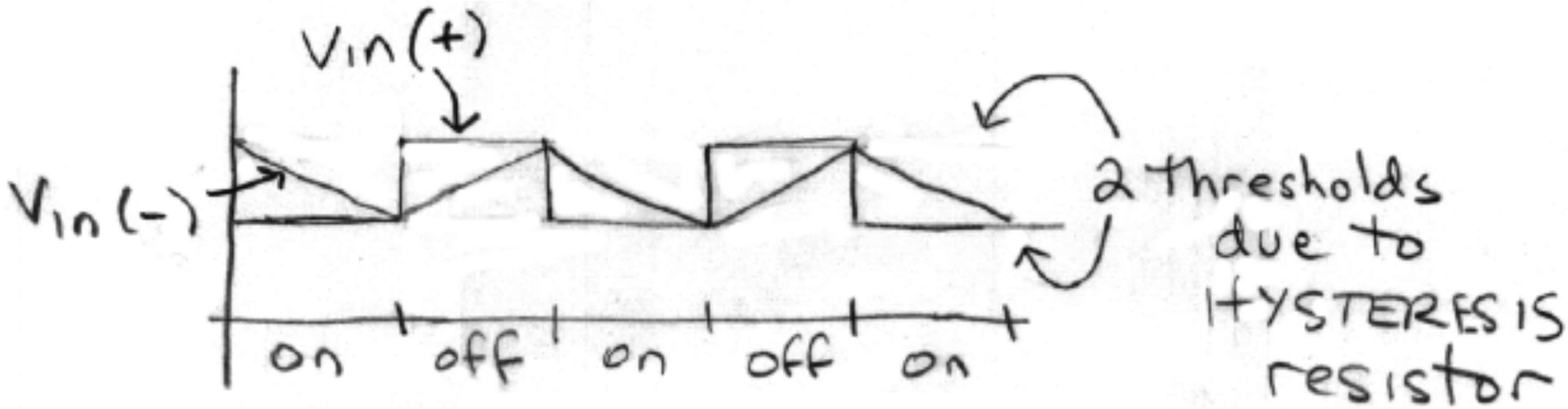




Oscillator

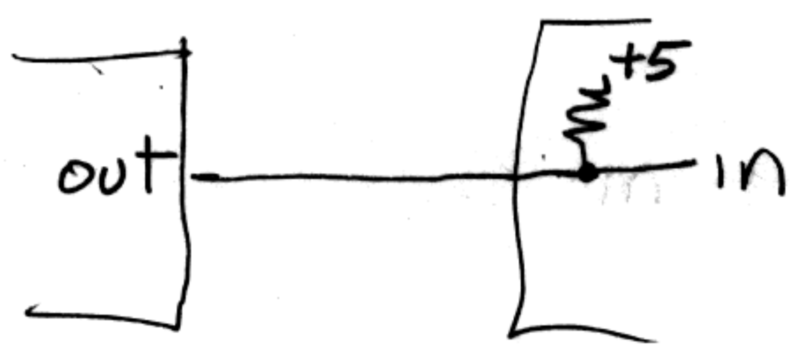
- Capacitor charges and discharges between two thresholds.
- Similar to the thermoregulator circuit, but with a capacitor as "memory" instead of heat.





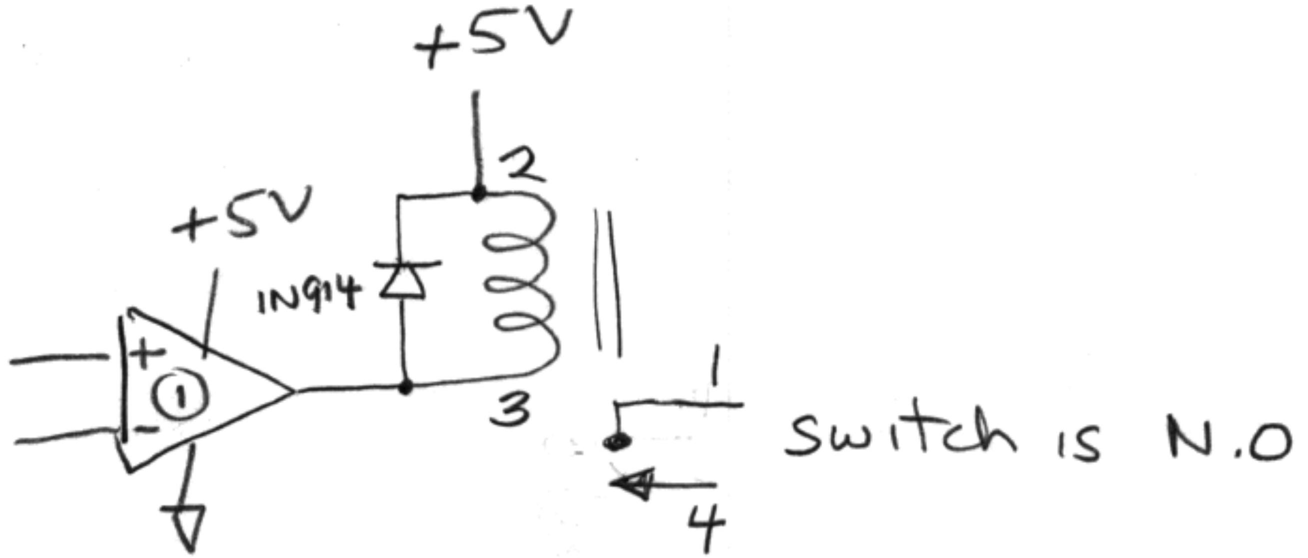
Reasons for open collector

- Digital inputs usually have an internal pull-up resistor.
- The open collector avoids needing to specify the “1” voltage (e.g. 5 V) for such inputs.
- The “0” voltage is always ground.
- Sub-systems do not need to share power supplies, just grounds.
- Two outputs can be connected together and either can pull the input to ground.

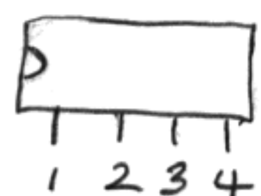


Comparator powering a relay

- Advantage of relay
  - Switch provides practically 0 to ∞ resistance
  - Complete isolation between coil and switch

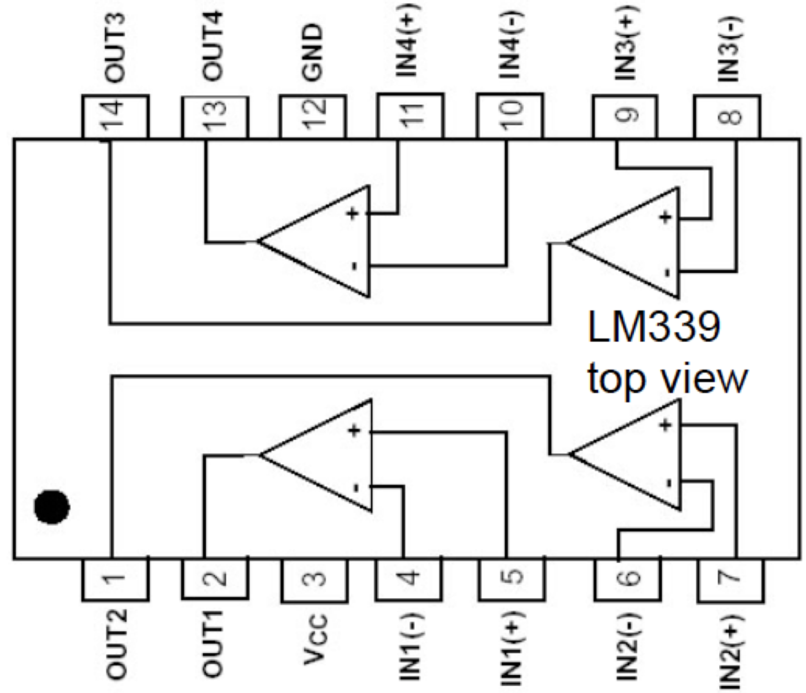


1 LM339
Relay, Digikey
HE206-ND (SPST)



1 Amp contacts, coil resistance 500 Ω
Diode for surge protection.

Our Comparator
the LM339



- Gain >10^6
  - practically infinite
- Input current 25 nA
  - practically infinite input impedance
- Response time 1 μS
  - very fast, generally slams all the way up or down
- Output 16 mA
  - maintains desired voltage up to this current (open collector)

LM339
other characteristics

- Power supply 2-36 V
  - Single-sided, can be simple battery.
- Quiescent supply current 0.8 mA
  - Current used within the comparator itself.
- Max output saturation voltage 1 V
  - When Open Collector output transistor is fully on
- Max offset voltage (input + to -) 3 mV
  - Maximum error in voltage comparison between inputs

Comparator vs. Full Op Amp

Comparator

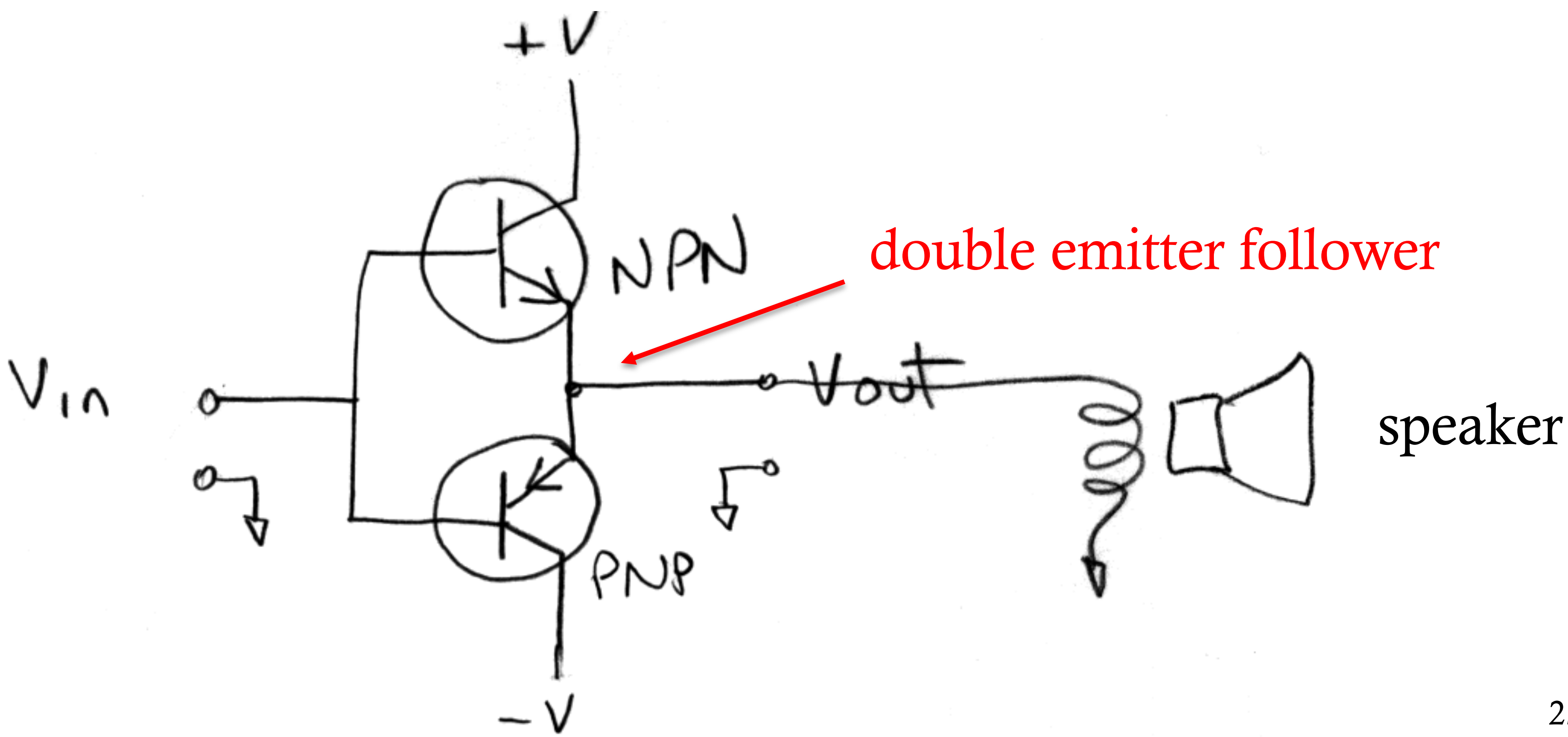
- output basically digital, binary question: which input voltage is higher?
- faster (response time)
- open collector output
- usually just + power with ground the lowest voltage

Full Op Amp

- output fully analog, basis for *linear systems with gain*.
- slower (slew rate)
- push-pull output
- usually +/– powers with ground in the middle between them

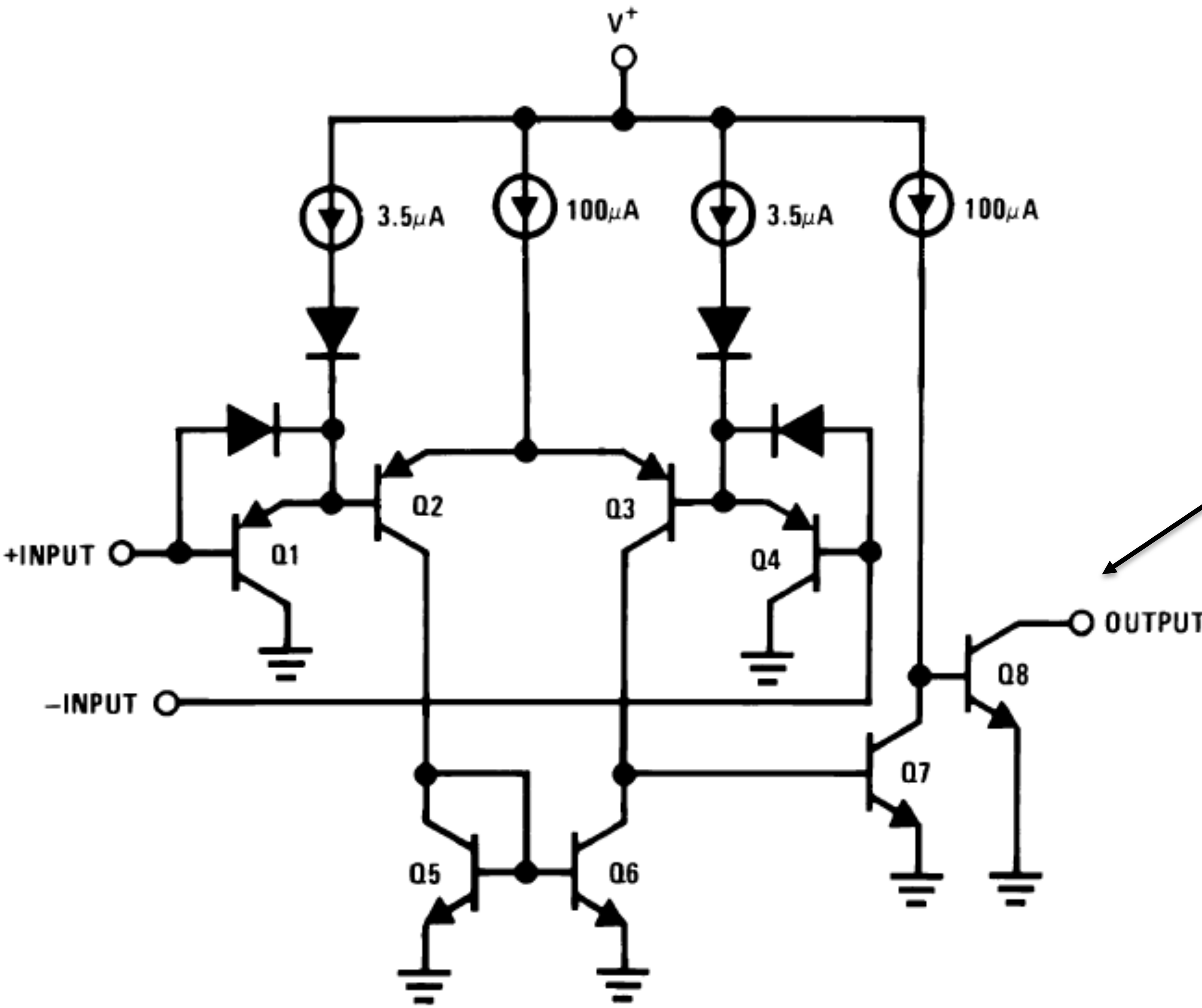
Push-Pull Output Stage

- Unlike the Open Collector output of a comparator, which can only *sink* current, the full Op Amp has a Push-Pull Output stage that can *sink* or *source* current,



Looking inside the Integrated Circuit (IC):

Comparator



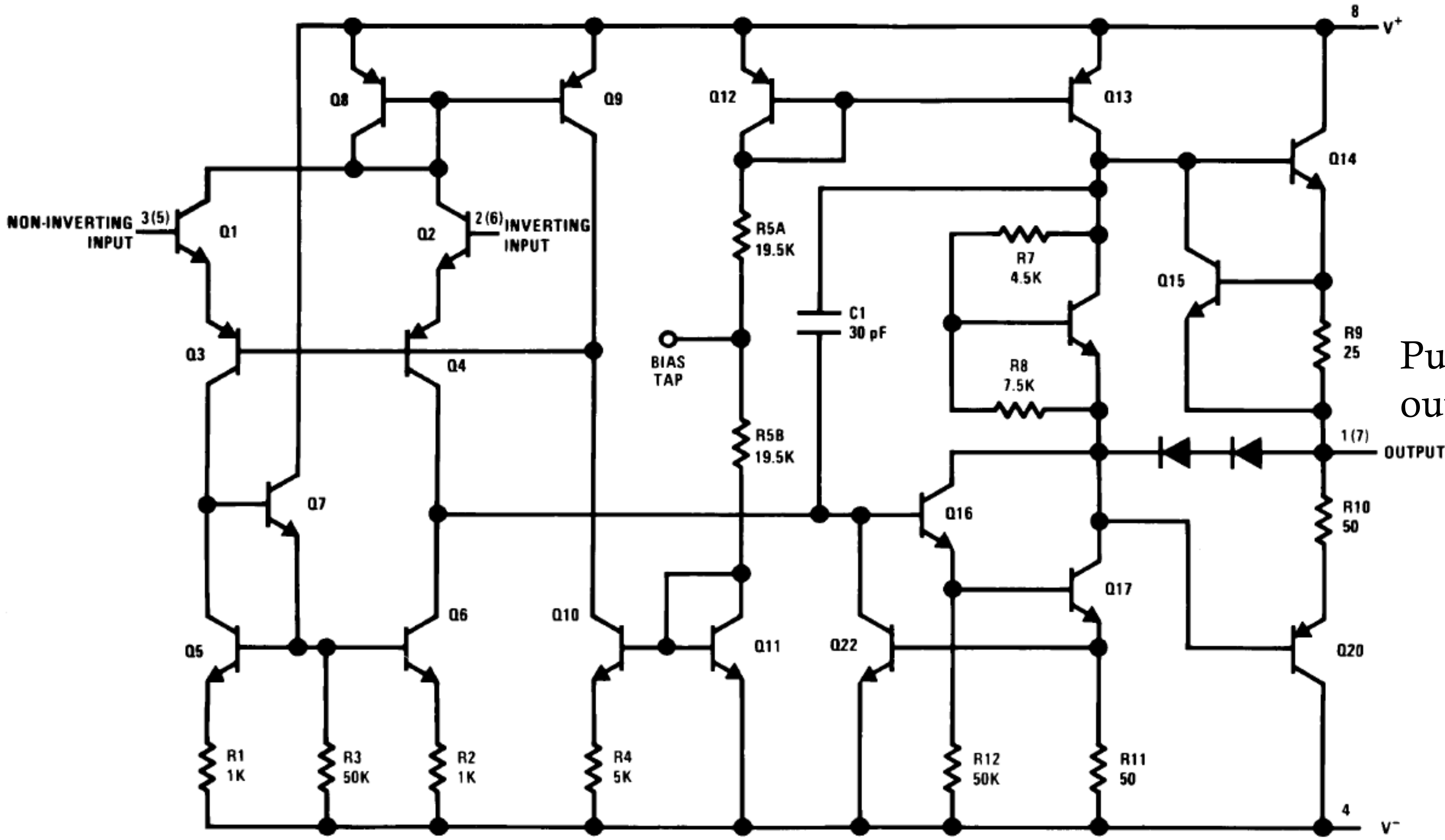
No resistors or capacitors.

Fast, simple, basically digital.

Open collector output

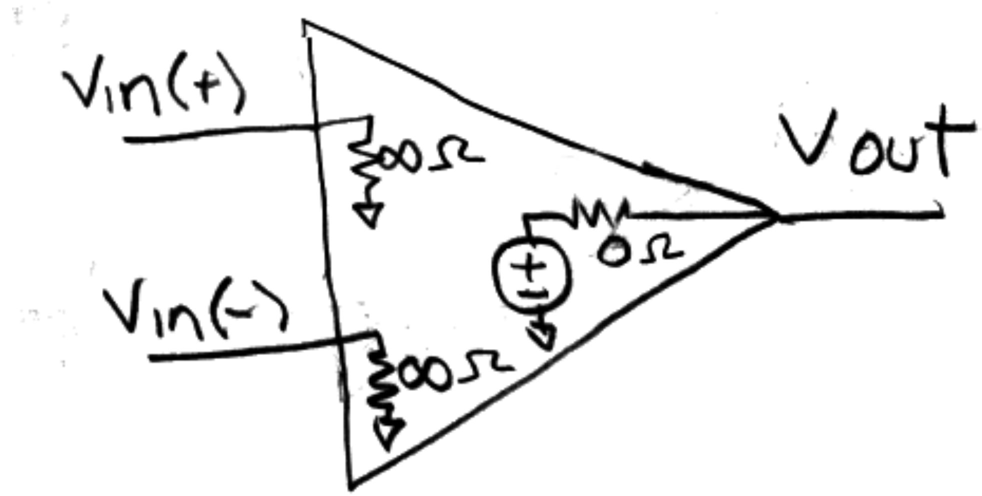
Full Operational Amplifier

Lots of resistors and capacitors.
Not as fast. Subtle. Analog.



Push-Pull output

Operational Amplifier

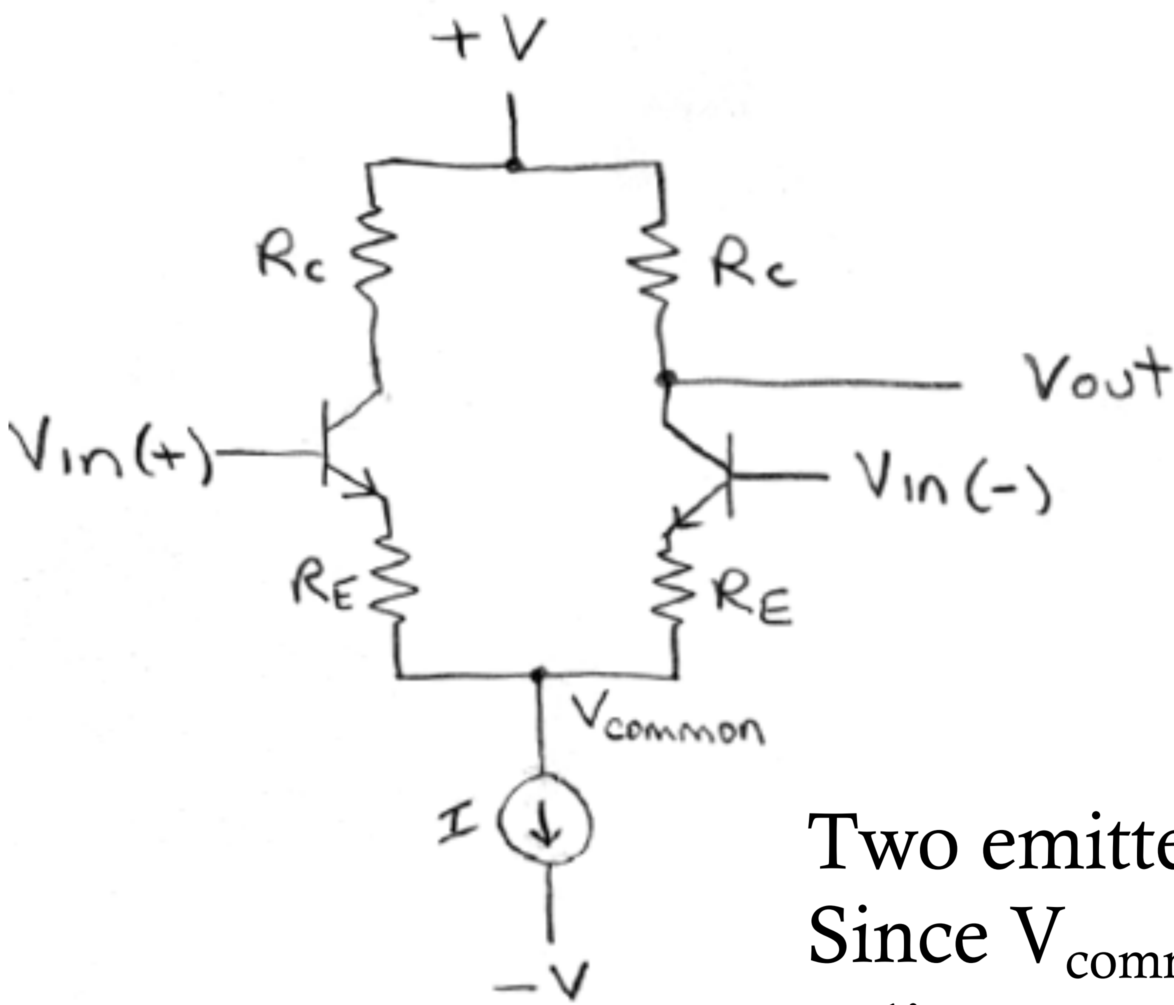


$$V_{\mathrm{out}} = A(V_{\mathrm{in}+} - V_{\mathrm{in}-}), \quad A \to \infty$$

Properties of Ideal Op Amp (review)

1. Infinite Input Impedance
2. Zero Output Impedance
3. Infinite Gain

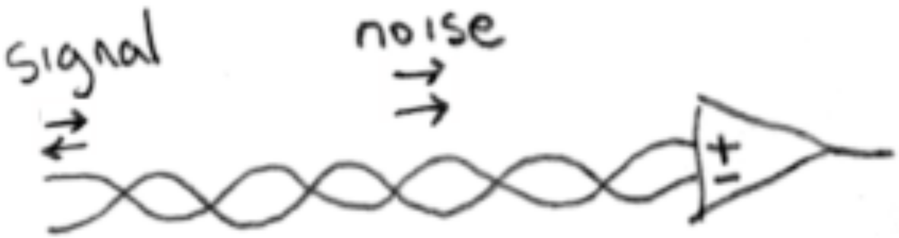
Differential Input



First stage in
Comparator
or Full Op Amp

Two emitter follower circuits. Since V_{common} floats, current I is split proportional to V_{in+} and V_{in-} with corresponding voltages across the two equal resistors R_C.

Differential amps reject noise



Twisted pair takes both wires through same noise-generating spatial electromagnetic field.

Signal is out of phase.
Noise is in-phase (common mode).

If gains are equal (single "gain" A)…

$$V_{\text{out}} = A(V_{\text{in}+} - V_{\text{in}-})$$

then noise exactly cancels.

However, in reality gains are unequal ($A_1 \neq A_2$):

$$V_{\text{out}} = A_1 V_{\text{in}+} - A_2 V_{\text{in}-}$$

Common Mode Rejection Ratio

How well does a real differential amp reject noise? Since noise is "common-mode," the measure is called Common Mode Rejection Ratio (CMRR).

$$V_{\text{out}} = A_1 V_{\text{in}+} - A_2 V_{\text{in}-}$$

$$\boxed{\text{CMRR} \triangleq \frac{1}{2}\frac{A_1 + A_2}{A_1 - A_2}}$$

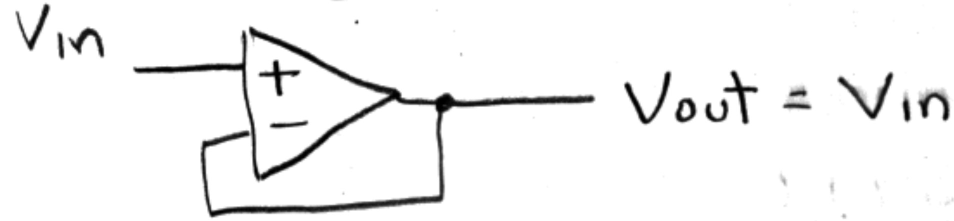
Goal is to make ($A_1 = A_2$), so that CMRR = ∞ and

$$V_{\text{out}} = A(V_{\text{in}+} - V_{\text{in}-})$$

Full Operational Amplifier

OP AMP CIRCUITS

VOLTAGE FOLLOWER also called a *buffer*

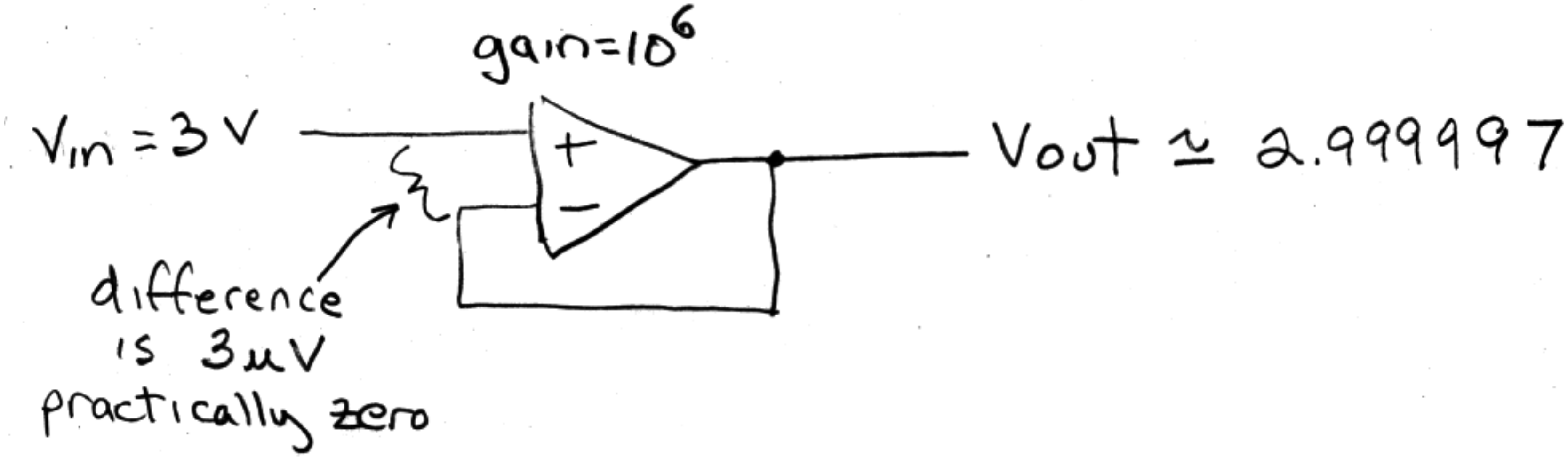


+ and − inputs always practically equal, if circuit properly designed and biased. The output will do what it needs to do to guarantee this.

In practice, set the inputs equal and then figure out what the output must be.

Let's look in detail just this once...

assume gain is 10^6, $V_{in} = 3V$



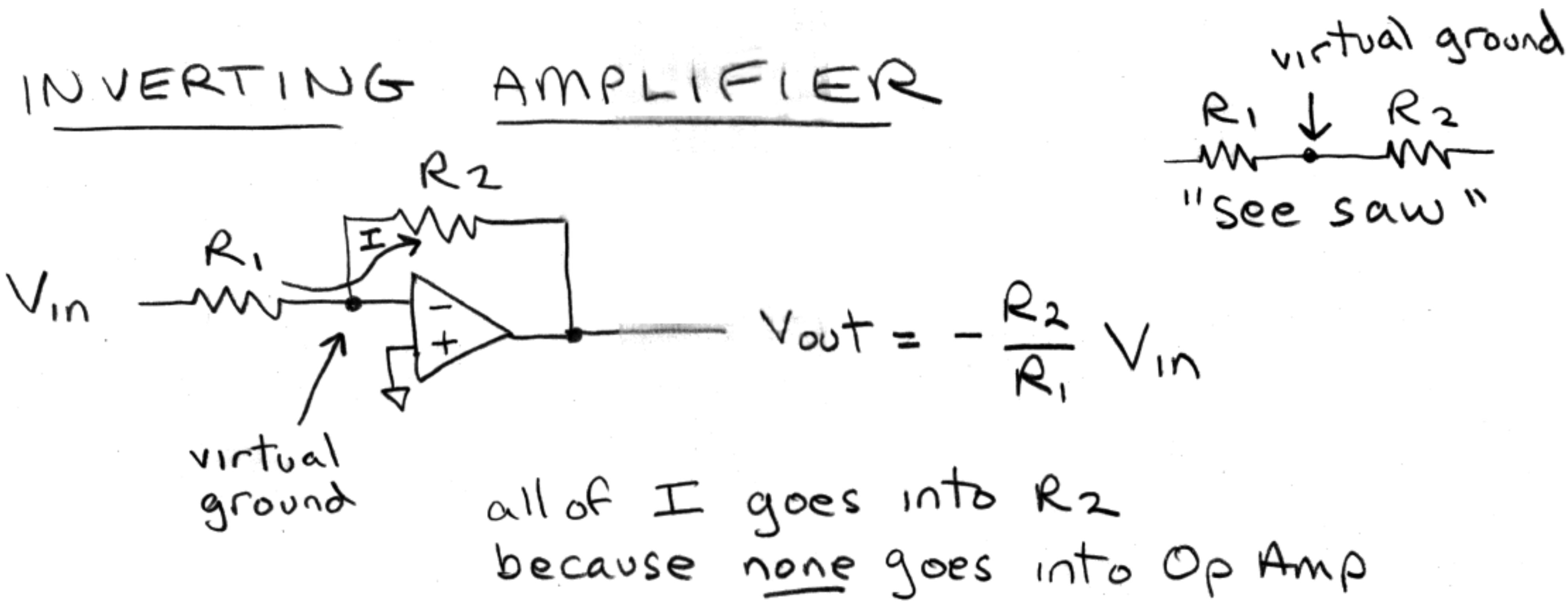
assume gain is $\sim\infty$, then to keep output finite (not plastered against the + or − power supply) the inputs must be practically equal.

Negative Feedback –

If output too high, it is caused to go lower, and visa versa

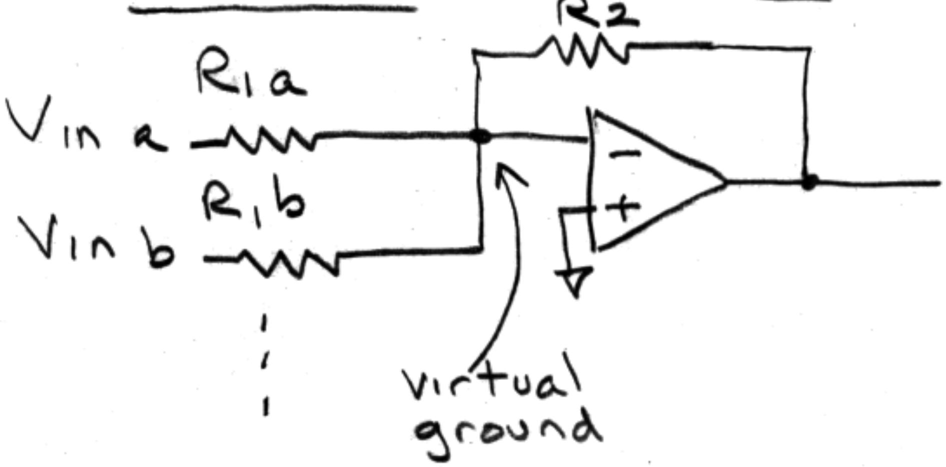
Virtual Ground – Inverting Amplifier

- When V_{in} is above ground, V_{out} goes below ground (and visa versa)
- Possible because we now have + and – power supplies



Inverting Adder

INVERTING ADDER (SUMMER)



$V_{out} = -\left[\frac{R_2}{R_{1a}} V_{in}a + \frac{R_2}{R_{1b}} V_{in}b + \ldots\right]$

currents entering virtual ground sum into R_2

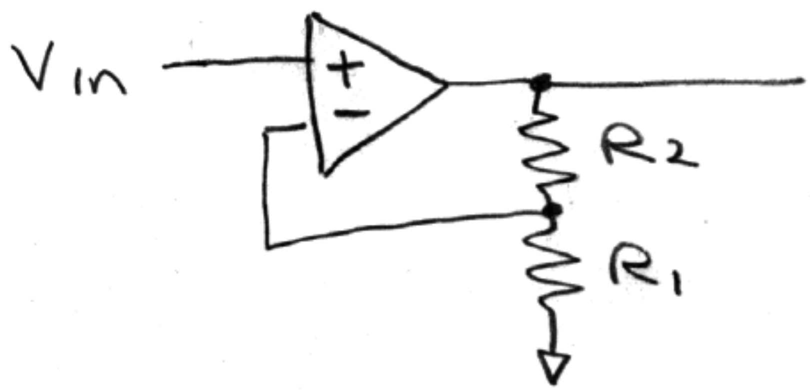
↑ these 2 circuits no longer have $\sim\infty$ input impedance as seen by V_{in}

Each input has its own gain. For example:

$$\text{Gain}_a = V_{out}/V_{in} = -R_2/R_{1a}$$

Non-Inverting Amplifier

NON-INVERTING AMPLIFIER

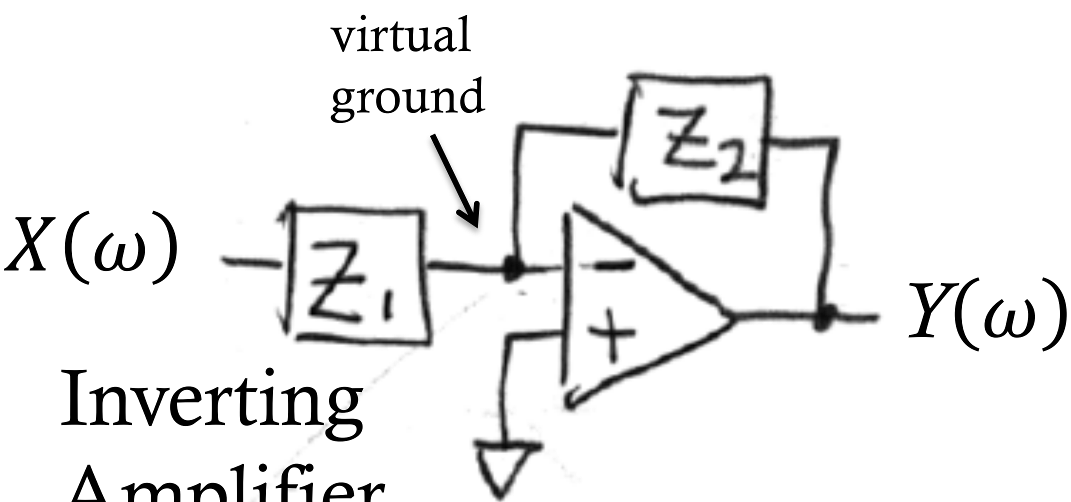


$$V_{out} = \frac{R_1 + R_2}{R_1} V_{in}$$

just the voltage divider equation

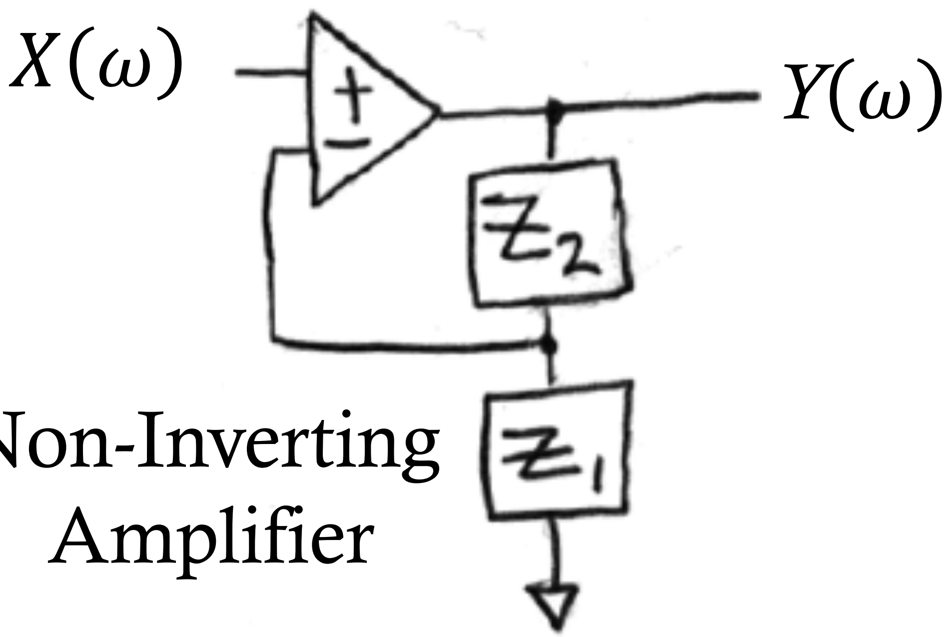
↑ this circuit <u>does</u> preserve ~∞ input impedance as seen by V_{in}

Op Amp Filter Circuits using Complex Impedance



Inverting
Amplifier

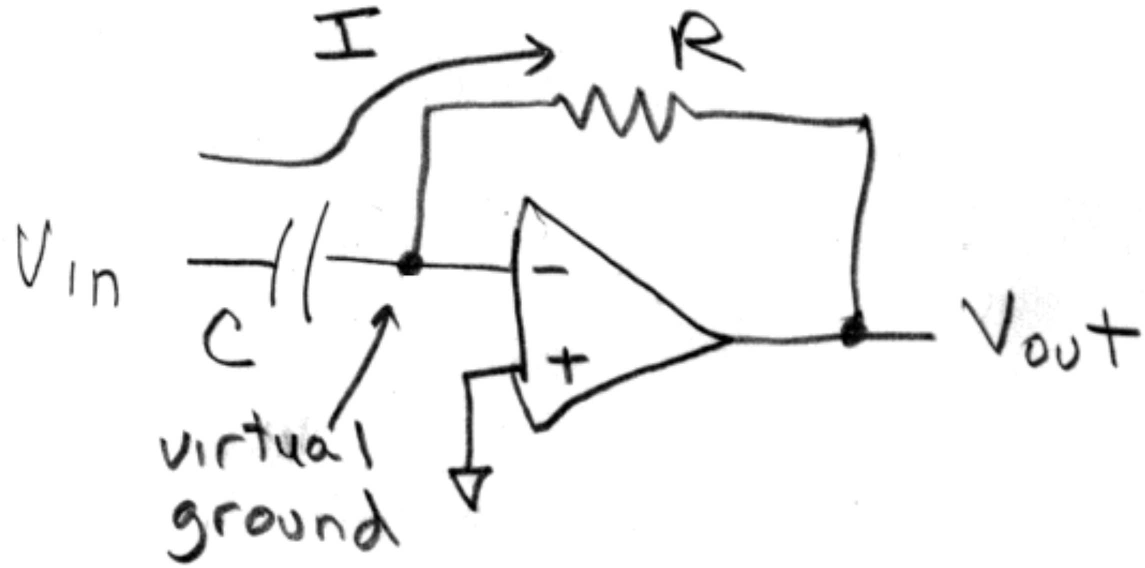
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = -\frac{Z_2}{Z_1}$$



Non-Inverting
Amplifier

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{Z_1 + Z_2}{Z_1}$$

Differentiator (favors *high* frequencies)



$$I = C\frac{dV_{in}}{dt}$$

$$V_{out} = -IR$$

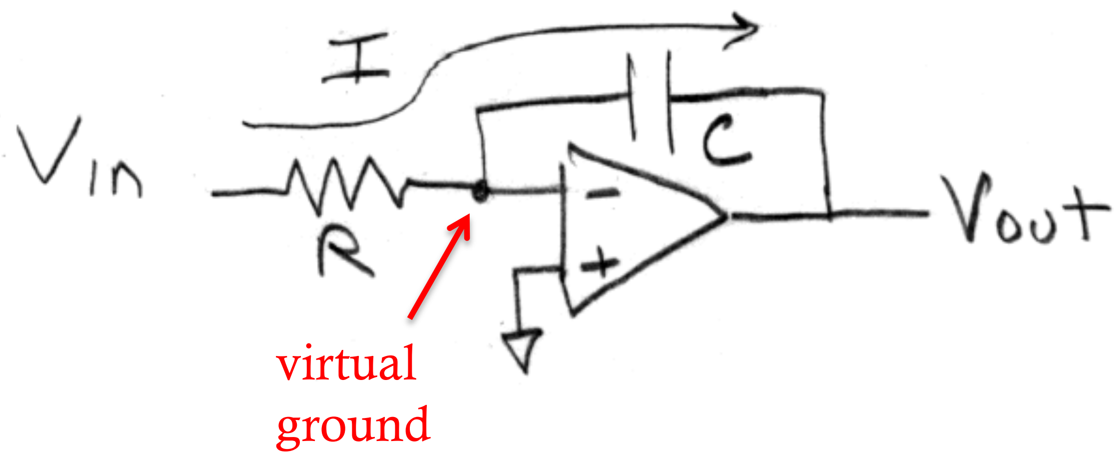
$$V_{out} = -RC\frac{dV_{in}}{dt}$$

equations say the same thing

or simply by applying the equation for $H(\omega)$ of the inverting amplifier

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = -\frac{Z_2}{Z_1} = -\frac{R}{{}^{1}/_{j\omega C}} = -j\omega RC$$

Integrator (favors *low* frequencies)



$$I = \frac{V_{in}}{R}$$

$$V_{out} = -\frac{1}{C}\int I\,dt$$

$$V_{out} = -\frac{1}{RC}\int V_{in}\,dt$$

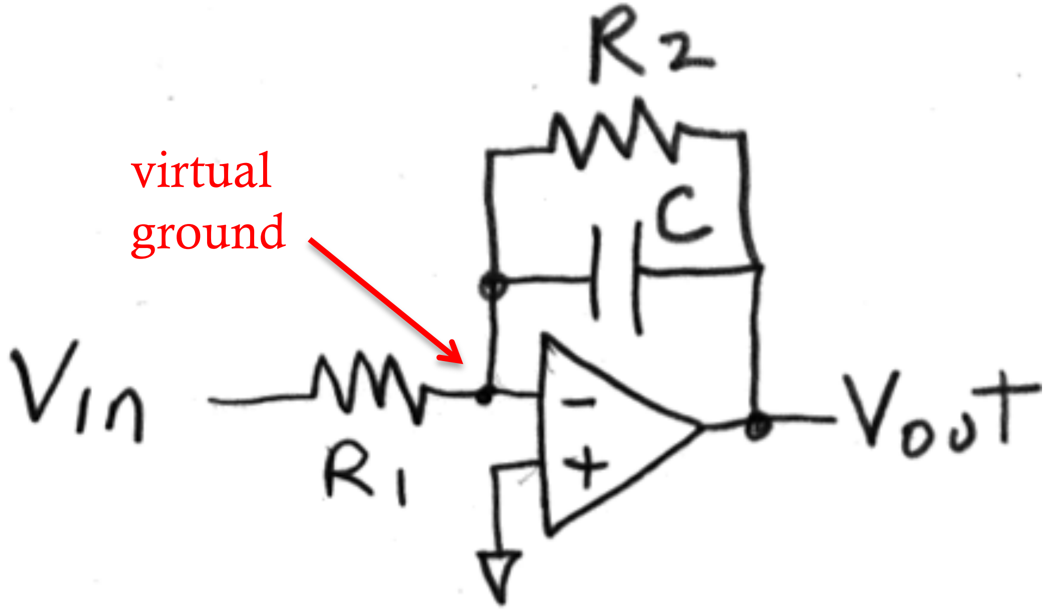
equations say
the same thing

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = -\frac{Z_2}{Z_1} = -\frac{{}^{1}/_{j\omega C}}{R} = -\frac{1}{j\omega RC}$$

$\rightarrow \infty$ at $\omega = 0$, DC

If V_{in} has non-zero average
$V_{out} \rightarrow \pm\infty$ given time

Add R_2 to prevent integrator from going to ∞ at DC



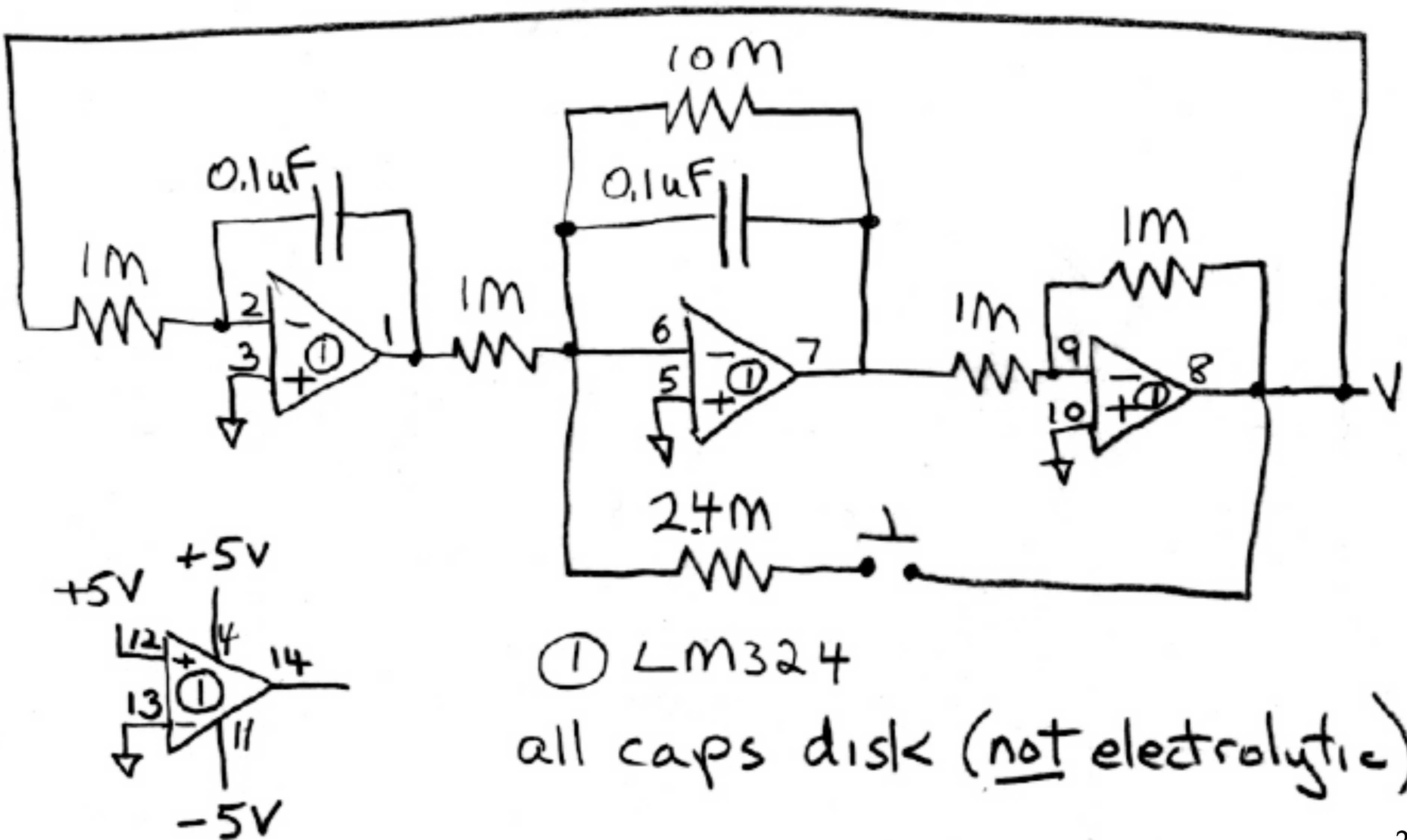
Derived realizing Z_2 is R_2 in parallel with C. Much simpler than equivalent differential equation!

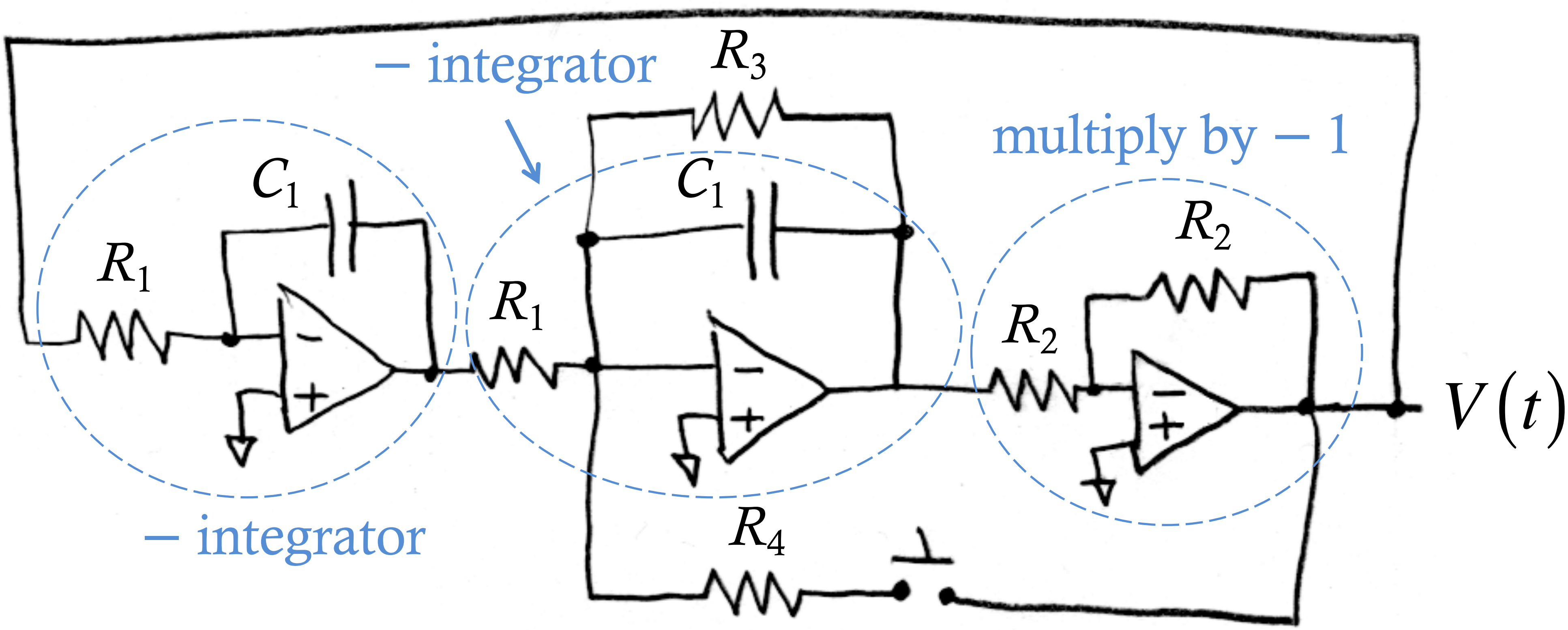
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1(1 + j\omega R_2 C)}$$

At DC ($\omega = 0$), capacitor disappears and gain $|H(\omega)| = -\frac{R_2}{R_1}$, not ∞.

Sinusoidal Oscillator

Op Amps can model any linear differential equation.
Basis of "Analog Computers" used before powerful digital computers.





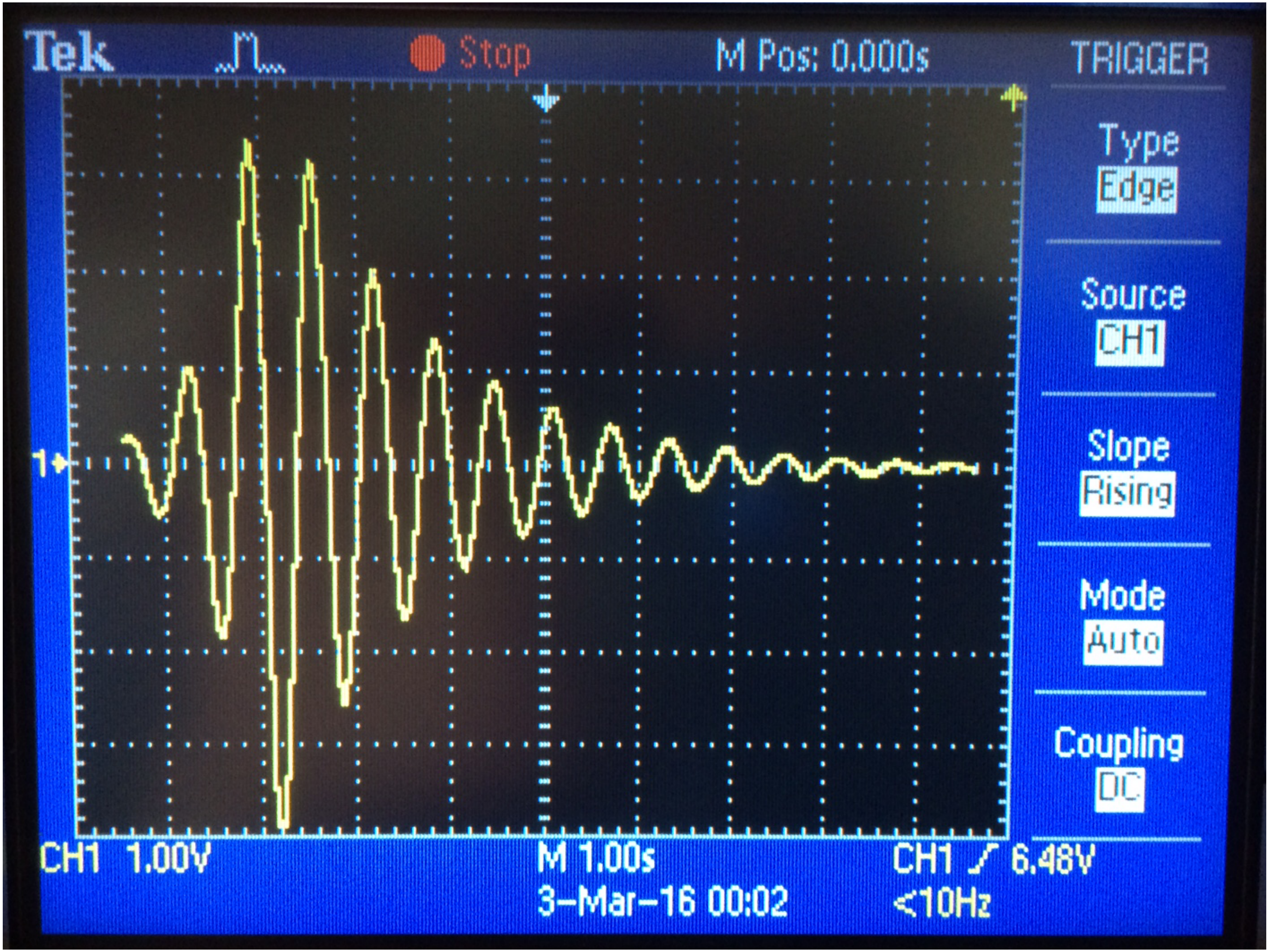
$$V(t) = -\int \frac{1}{R_1C_1}\left(\int \frac{1}{R_1C_1} V(t)\,dt\right) dt$$

$$\frac{d^2V(t)}{dt^2} = -\left(\frac{1}{R_1C_1}\right)^2 V(t)$$

$V(t)$ is sinusoid $\omega = \dfrac{1}{R_1C_1}$

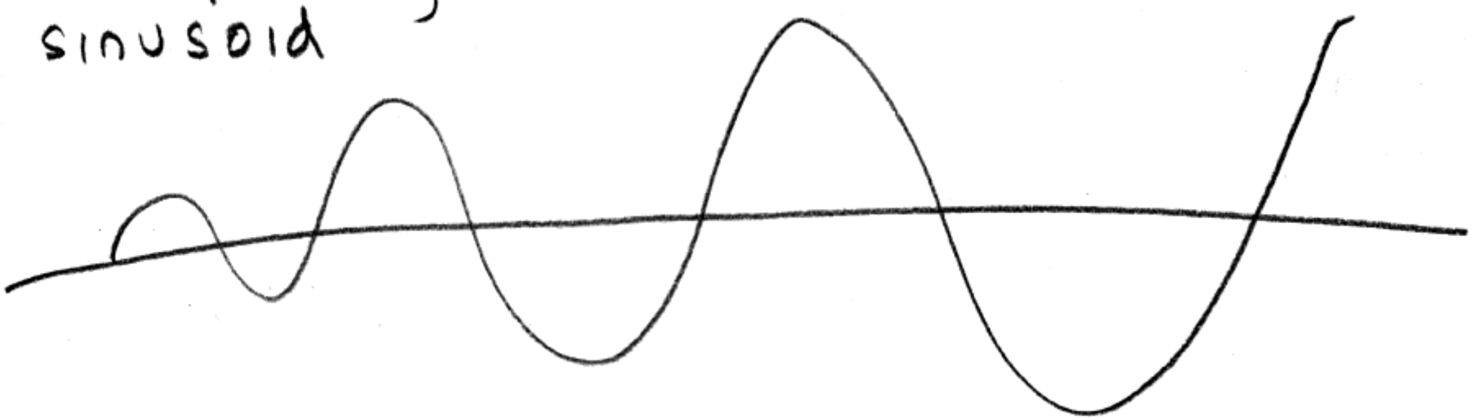
Amplitude of sinusoid determined by

$e^{-\frac{t}{R_3C_1}}$ and $e^{+\frac{t}{R_4C_1}}$


Tek
Stop
M Pos: 0.000s
TRIGGER
Type
Edge
Source
CH1
Slope
Rising
Mode
Auto
Coupling
DC
CH1 1.00V
M 1.00s
3-Mar-16 00:02
CH1 ⁄ 6.48V
<10Hz

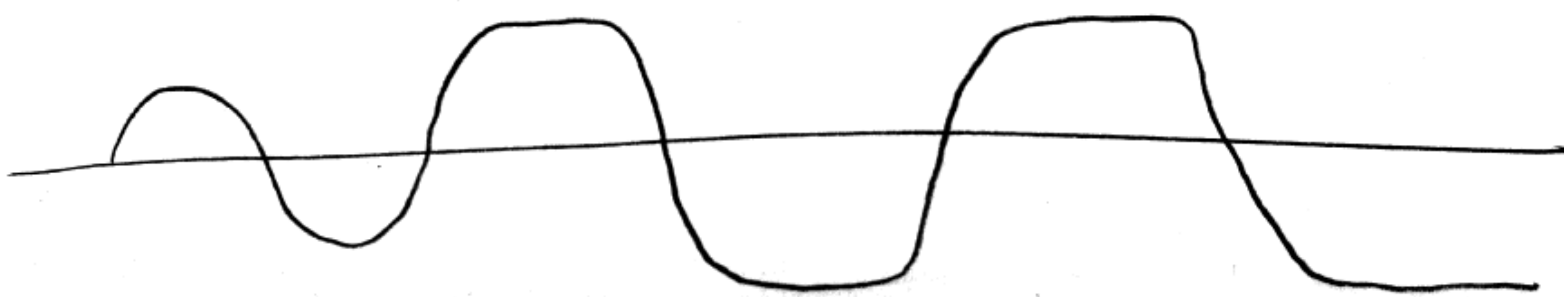
Why doesn't oscillator keep expanding?

in linear system
ever expanding
sinusoid



see movie

in non-linear (actual) system
becomes limited and stable

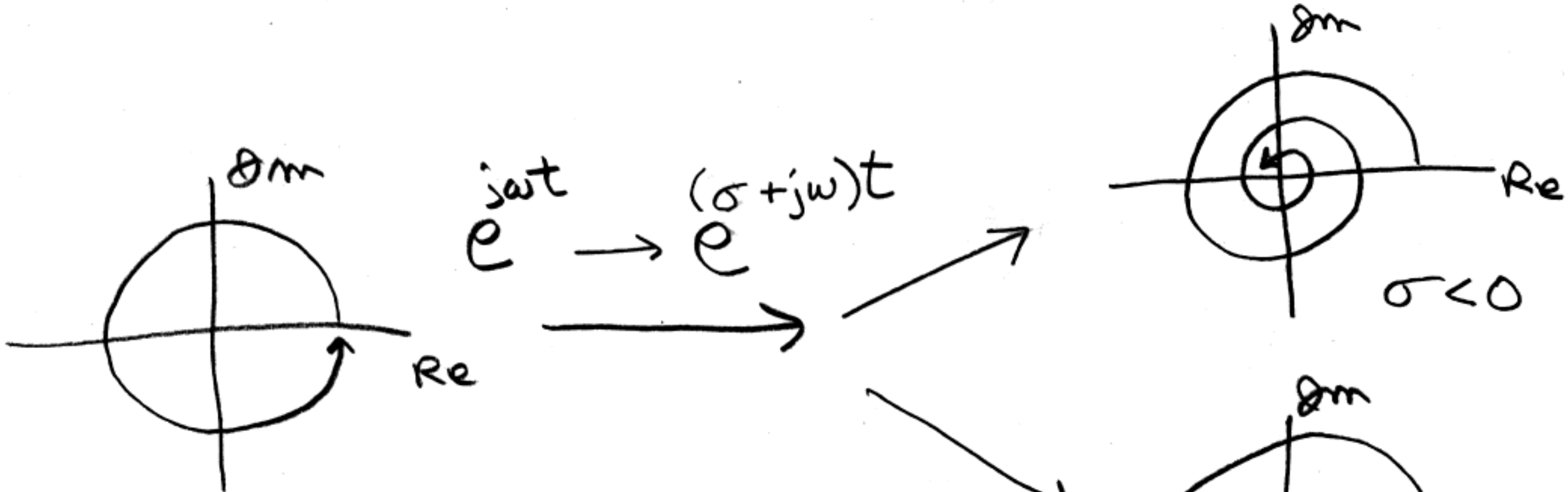


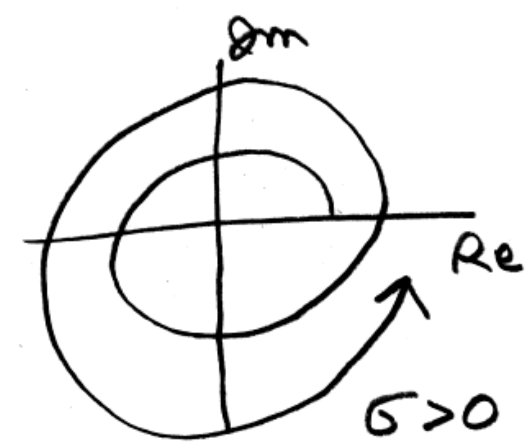
Laplace adds a real component σ to the phasor

$$e^{\sigma}e^{j\omega} = e^{(\sigma+j\omega)}$$

FOURIER $\longrightarrow$ LAPLACE

(ANY STABLE SIGNAL) (ANY LINEAR DIFF. EQ.)





- We use a new variable, "s"
- $s = \sigma + j\omega$
- Basis function becomes e^{st}

Recall the Fourier Transform

Applies to any *finite* signal (not just periodic)

Fourier Transform

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\, dt$$

Inverse Fourier Transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t}\, d\omega$$

Now becomes Laplace Transform

Applies to *any* signal (not just finite),
any linear differential equation.

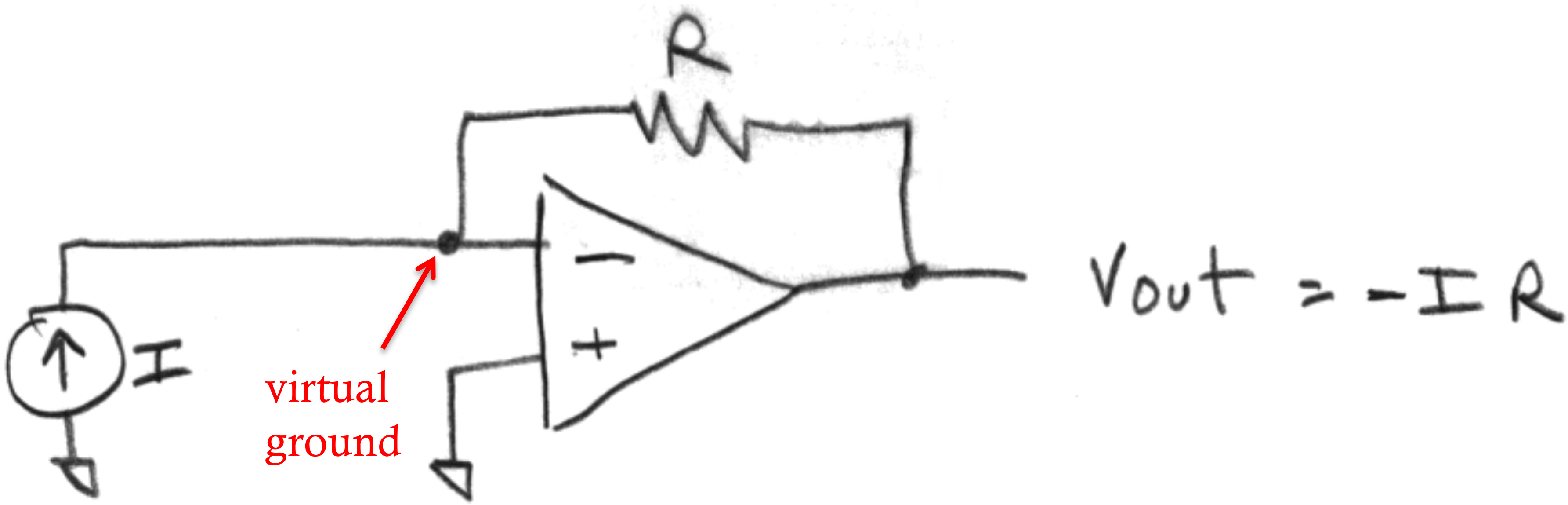
Laplace Transform

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

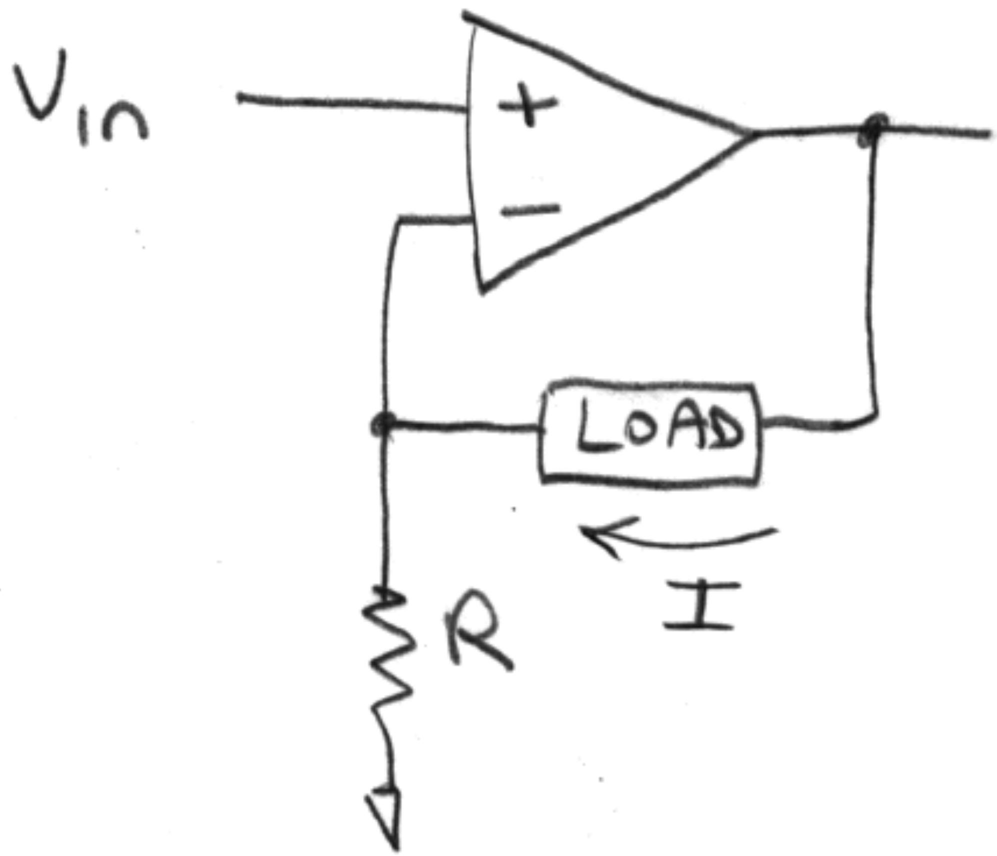
Inverse Laplace Transform

$$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{+st}ds$$

CURRENT TO VOLTAGE CONVERTER



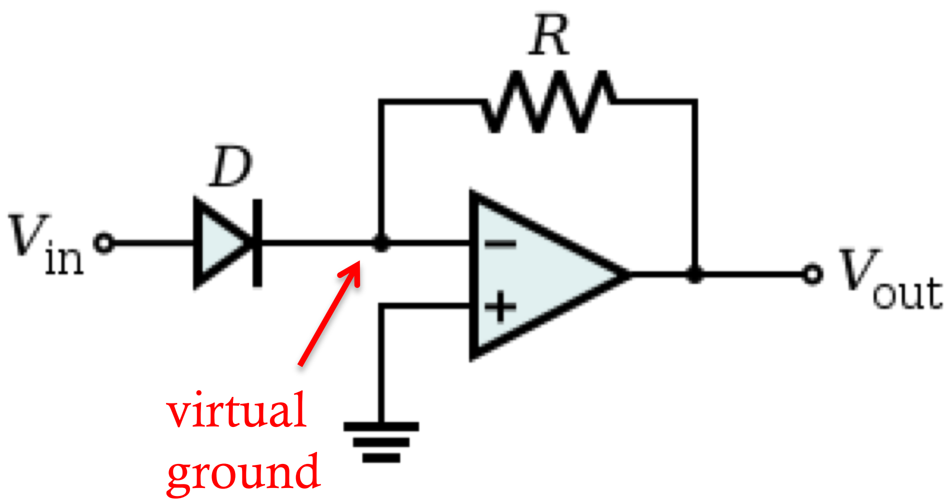
CURRENT SOURCE
(VOLTAGE TO current converter)



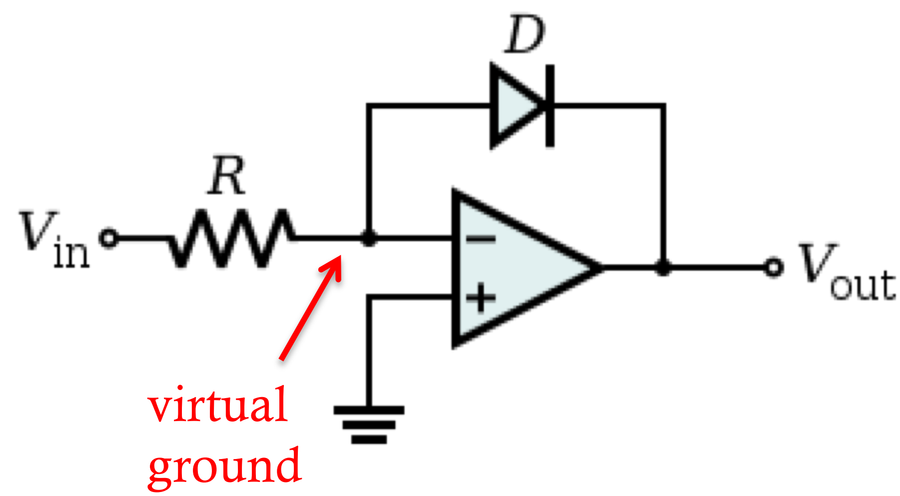
$$I = \frac{V_{in}}{R}$$

Non-linear Op Amp circuits with diodes

Exponential Amp



Log Amp



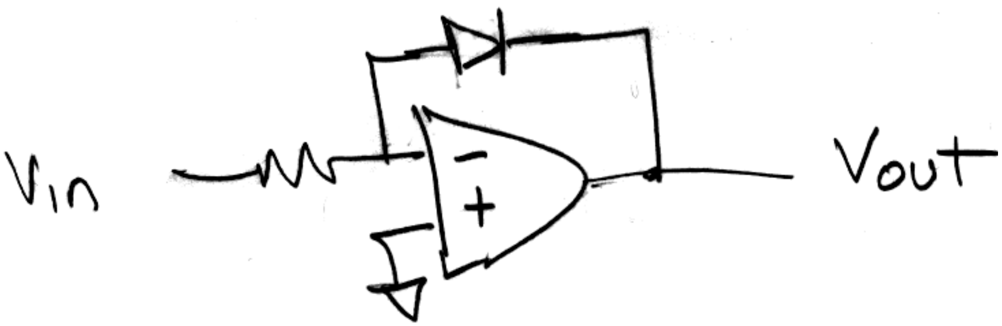
- Because current is exponential of voltage in diode.
- Now can multiply signals by taking log of each, then add them and take exponential.

https://en.wikibooks.org/wiki/Electronics/Electronics_Formulas/Op_Amp_Configurations

Log amp

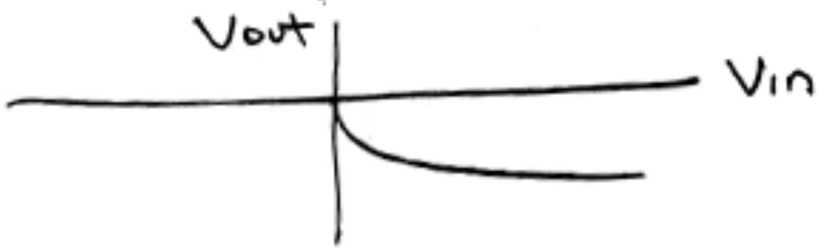
$V_{out} = -\ln V_{in}$

(with some constants)

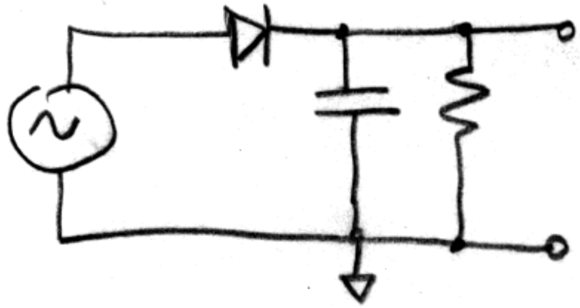


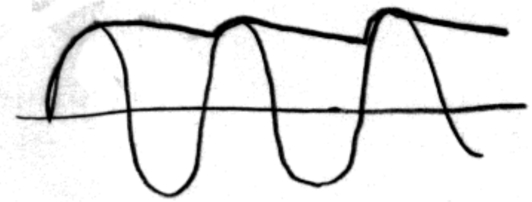
recall that diode behaves exponentially

thus feedback produces



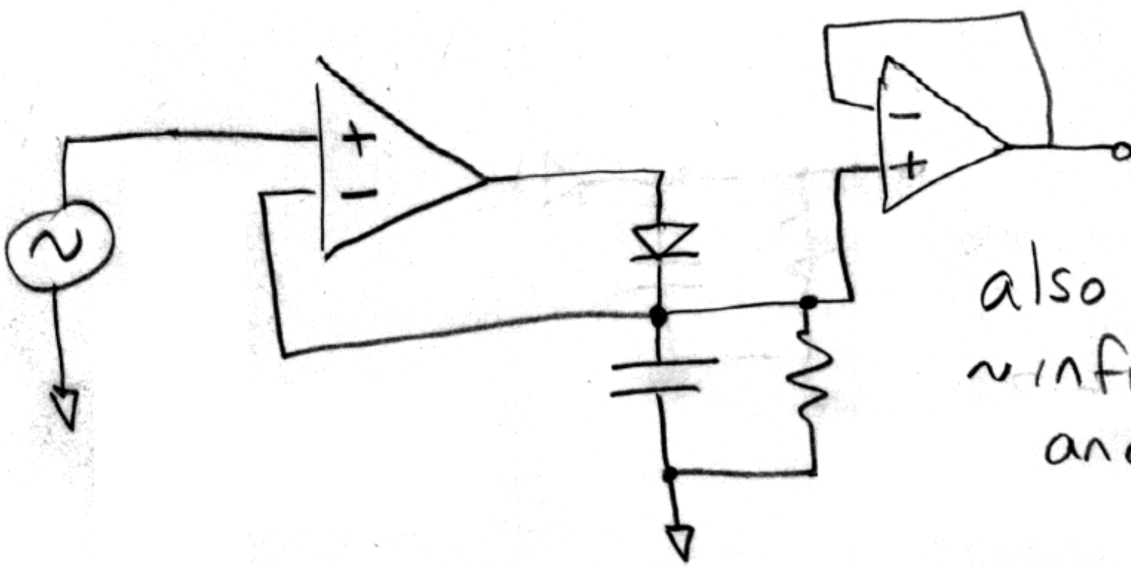
Peak Detectors





"demodulator" for Amplitude Modulation

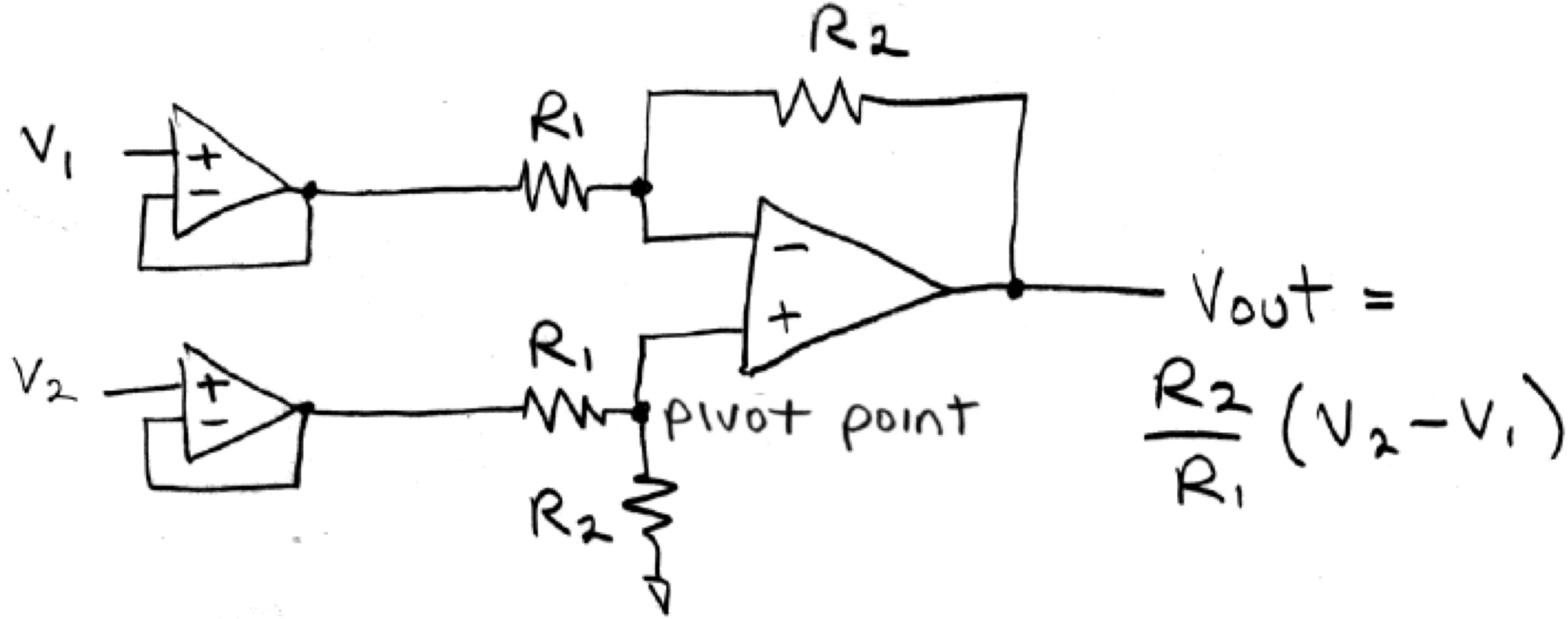
How to remove diode drop.



also gives system
~ infinite input impedance
and no output impedance

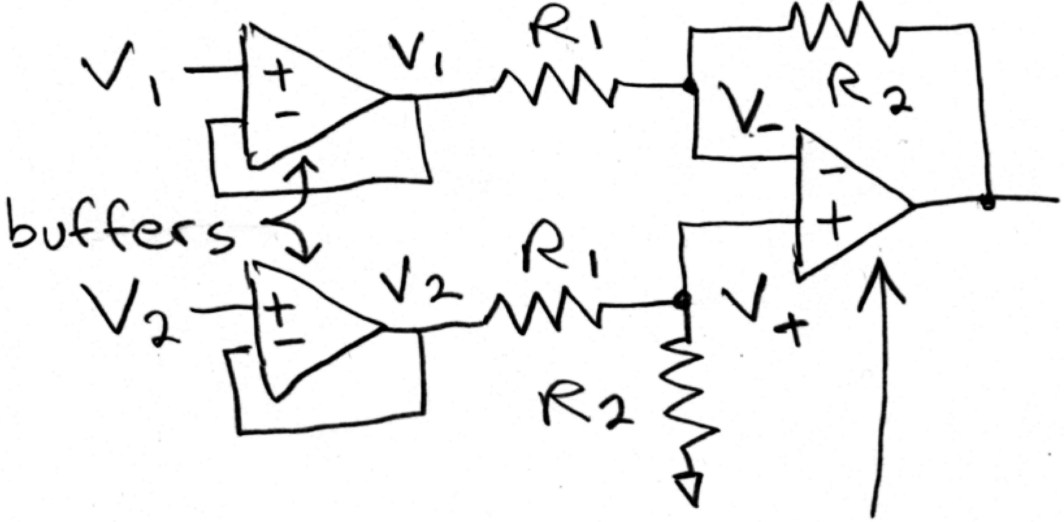
Multi Op Amp Circuits

Differential (Instrumentation) Amplifier



- Voltage followers used to provide "infinite" input impedance
- Finite gain determined by R_2/R_1
- Differential good for rejecting noise (CMRR), assuming matching resistors are used.
- Instead of virtual ground "pivot point" is set to $R_2/(R_1+R_2)$

Proove Differential Amplifier



$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

we know inputs of this op amp are equal

By <u>superposition</u> $V_- = V_1 \frac{R_2}{R_1+R_2} + V_{out} \frac{R_1}{R_1+R_2}$

setting $V_{out} = 0$ setting $V_1 = 0$

also, we know $V_+ = V_2 \frac{R_2}{R_1+R_2}$

Since $V_- = V_+$

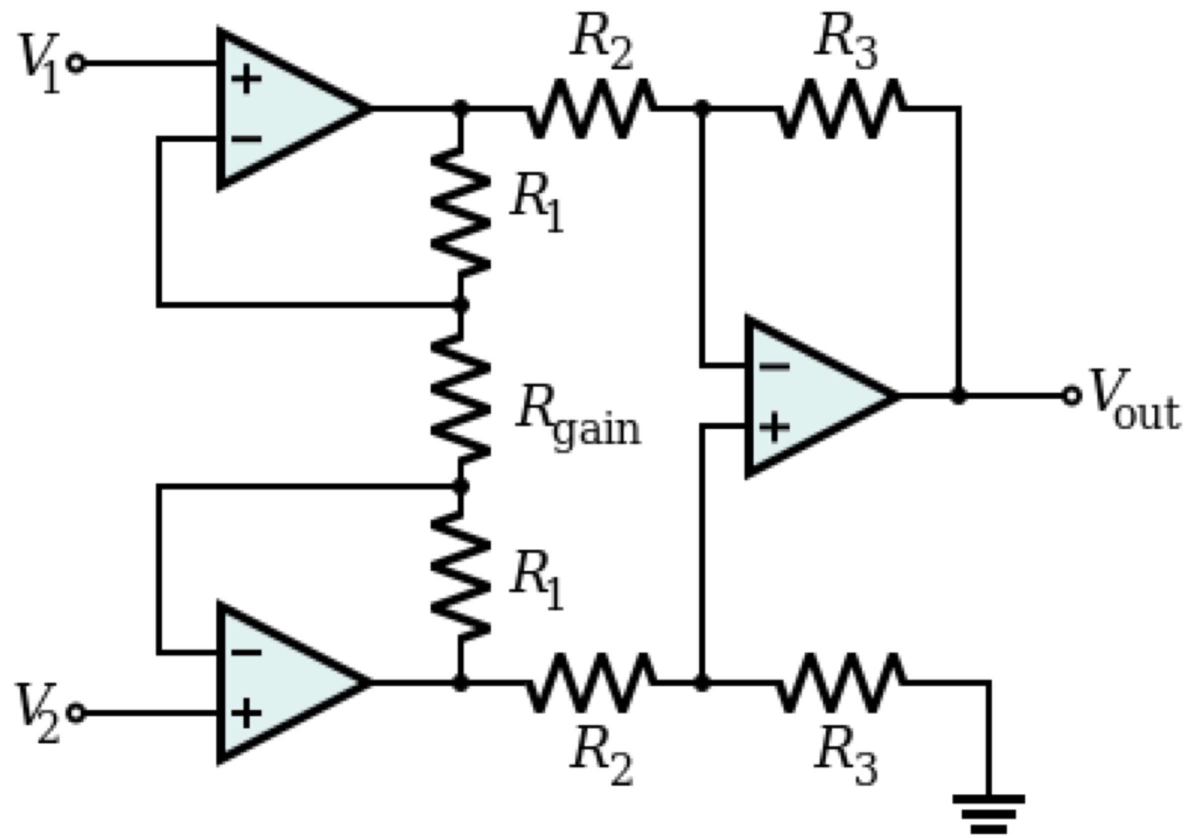
$$V_1 \frac{R_2}{\cancel{R_1+R_2}} + V_{out} \frac{R_1}{\cancel{R_1+R_2}} = V_2 \frac{R_2}{\cancel{R_1+R_2}}$$

$$V_1 R_2 + V_{out} R_1 = V_2 R_2$$

$$V_{out} = (V_2 - V_1) \frac{R_2}{R_1}$$ Q.E.D.

Actual Instrumentation Amplifier

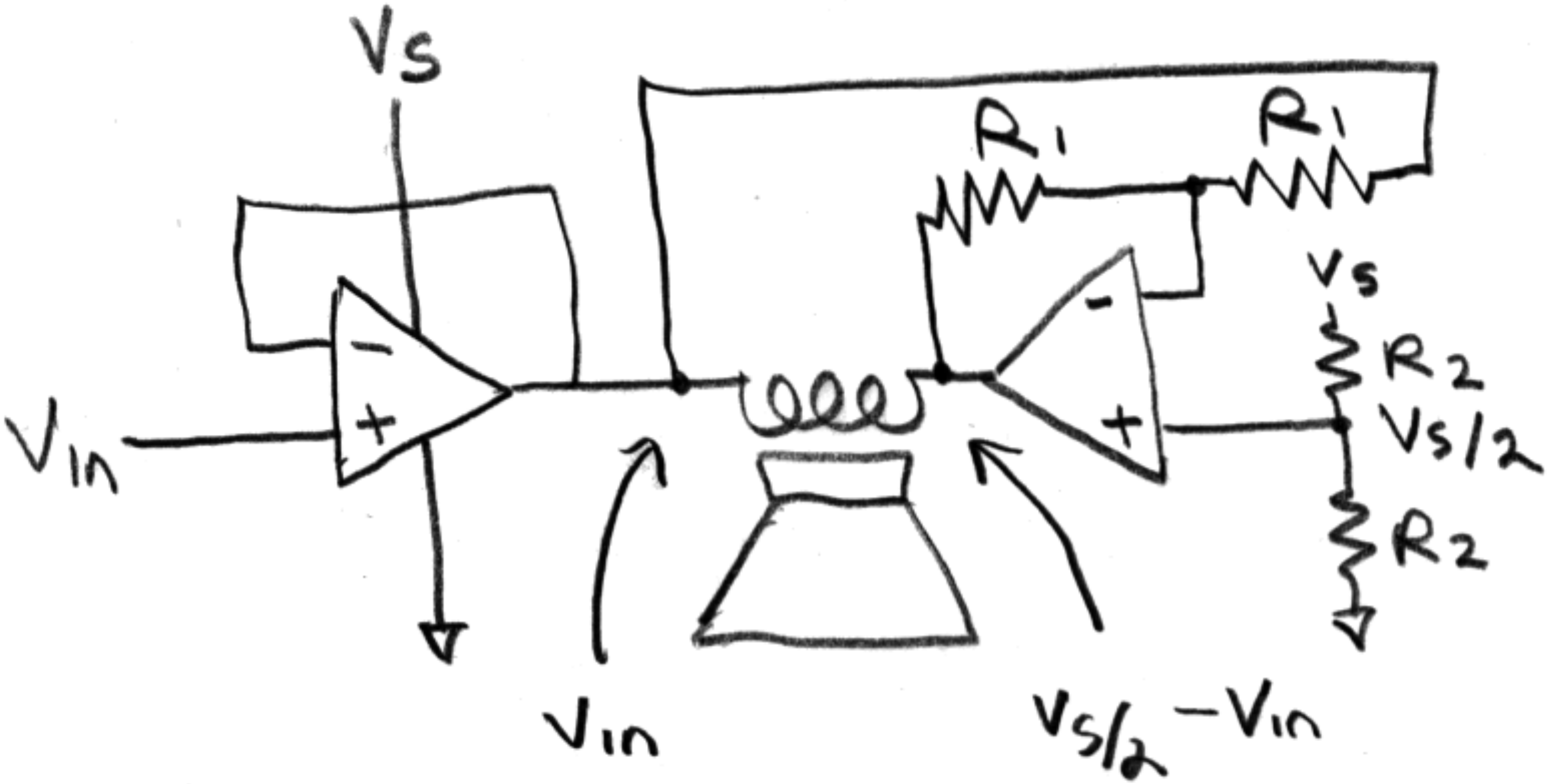
- Very high input impedance
- Gain controlled by a single resistor R_{gain}
- Single IC with R_{gain} only external resistor



$$\frac{V_{\text{out}}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{\text{gain}}}\right)\frac{R_3}{R_2}$$

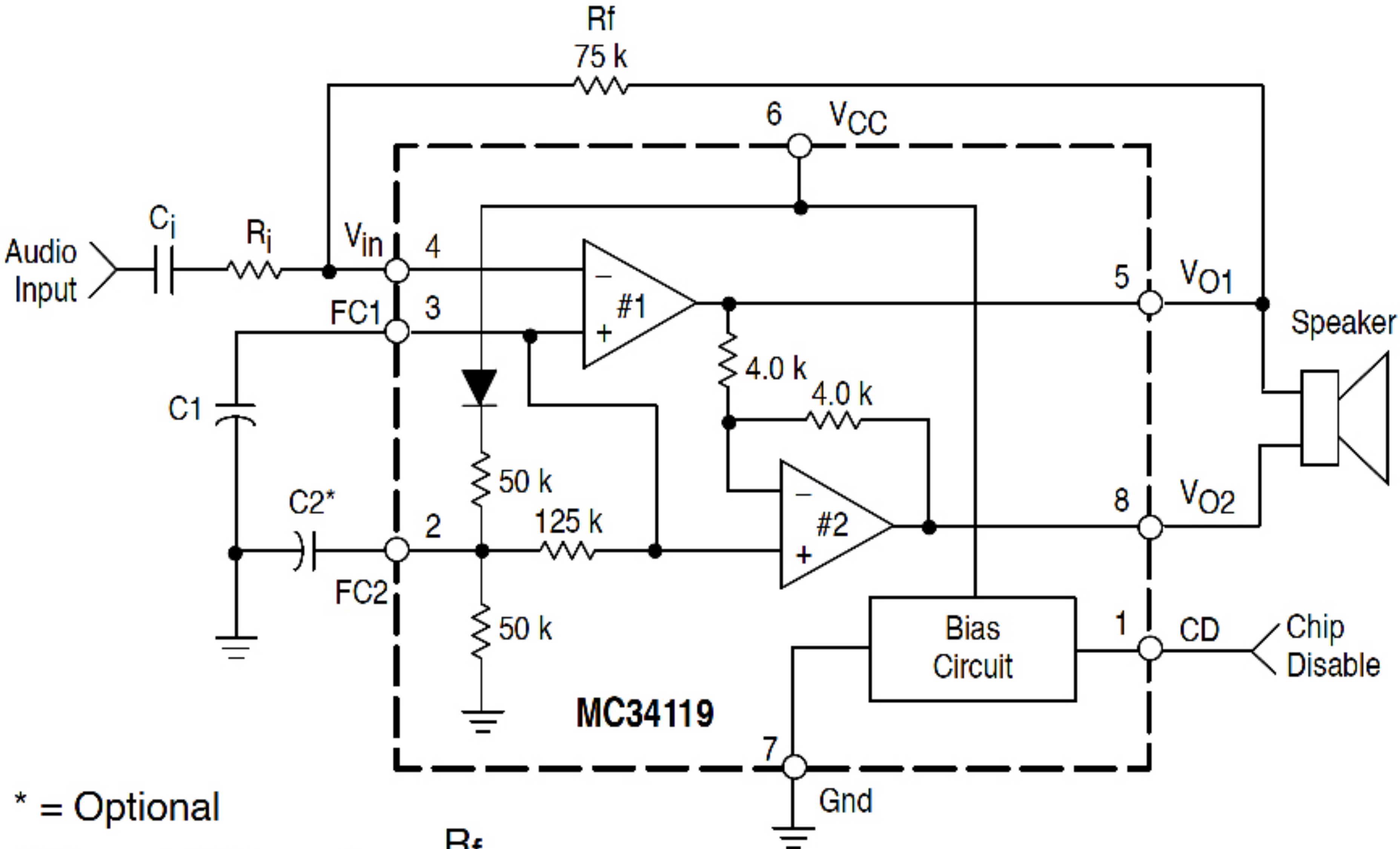
(e.g. AD623 from Analog Devices, Inc.)

H-Bridge Amplifier (in audio and elsewhere)



Push-Pull through speaker, with single-sided power supply, around pivot point Vs/2.

H-Bridge Audio Amp IC (MC34119) from Lab 4



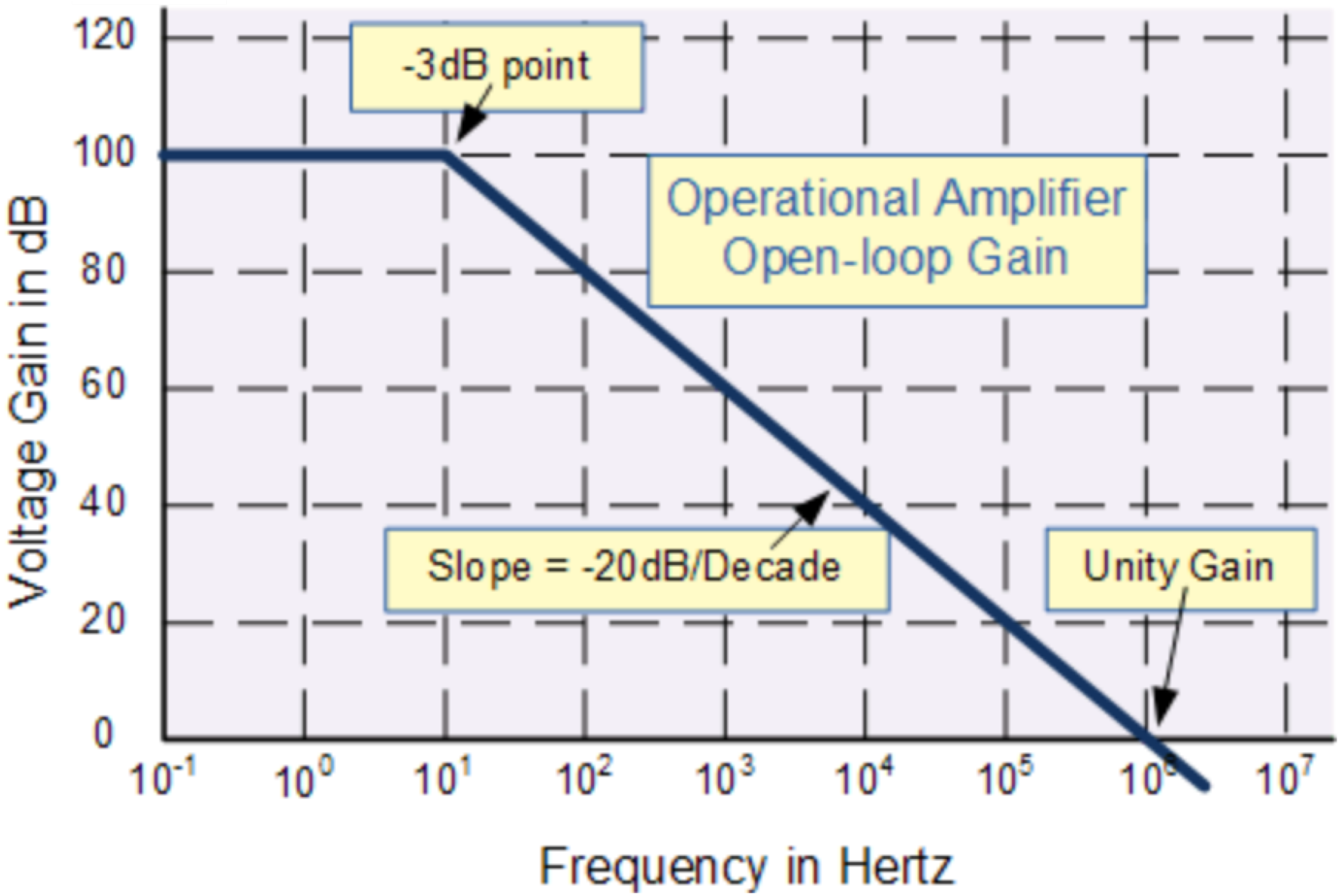
* = Optional

Differential Gian = 2 x $\frac{R_f}{R_i}$

This device contains 45 active transistors.

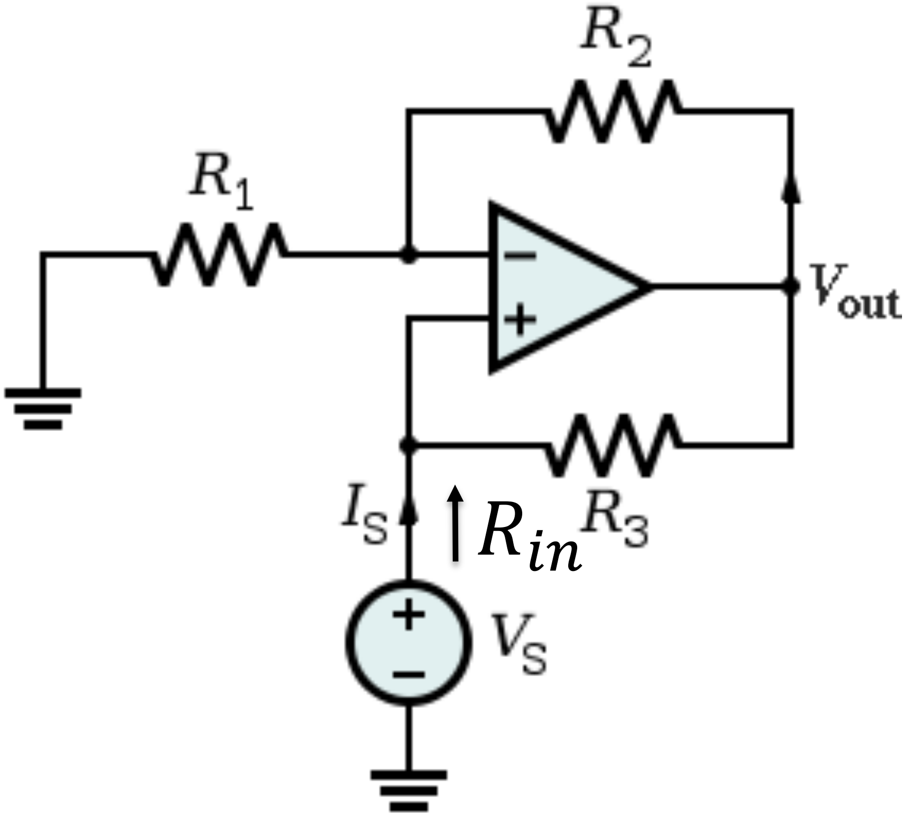
Real Op Amps: Open Loop Gain A vs. Frequency

where $V_{out} = A\left(V_{in+} - V_{in-}\right)$



http://www.electronics-tutorials.ws/opamp/opamp_1.html

Negative Resistance



- Circuit presents an effective *negative* input resistance R_{in}=V_S/R_S to signal generator V_S
- Proof: op amp inputs are equal,

$$V_S = V_{out}\frac{R_1}{R_1+R_2}$$

- Also,

$$V_{out} = V_S - I_S R_3$$

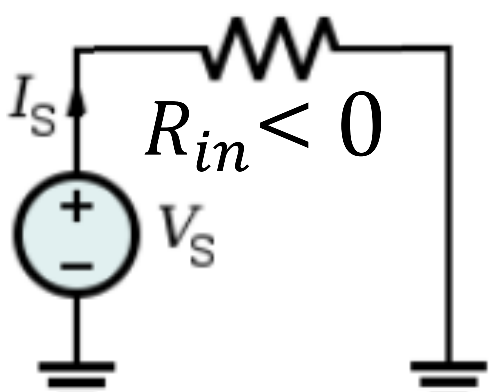
- Combining these yields,

$$V_S\left[\frac{R_1+R_2}{R_1} - \frac{R_1}{R_1}\right] = -I_S R_3$$

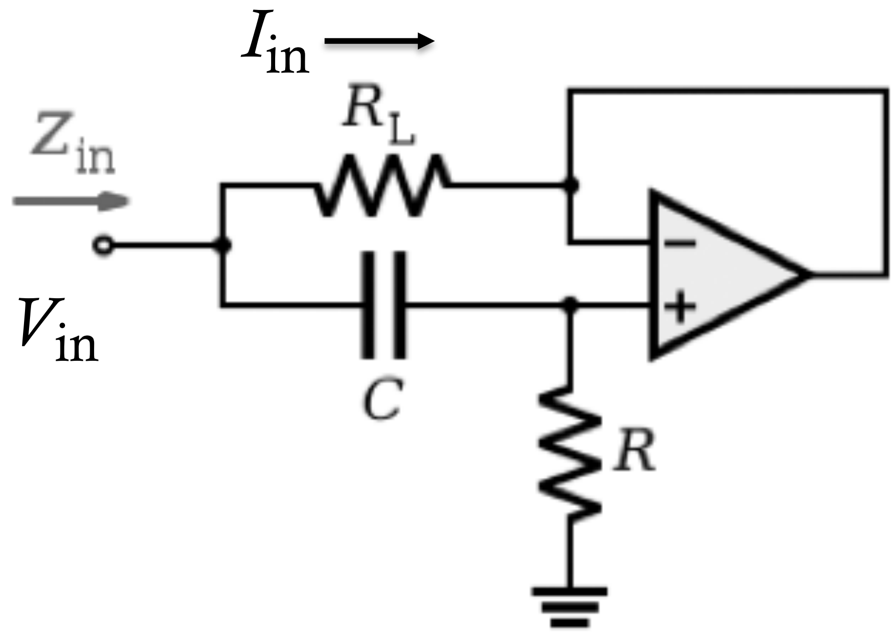
$$R_{in}=\frac{V_S}{I_S} = -R_3\,\frac{R_1}{R_2}$$

Negative Resistance

Equivalent Circuit

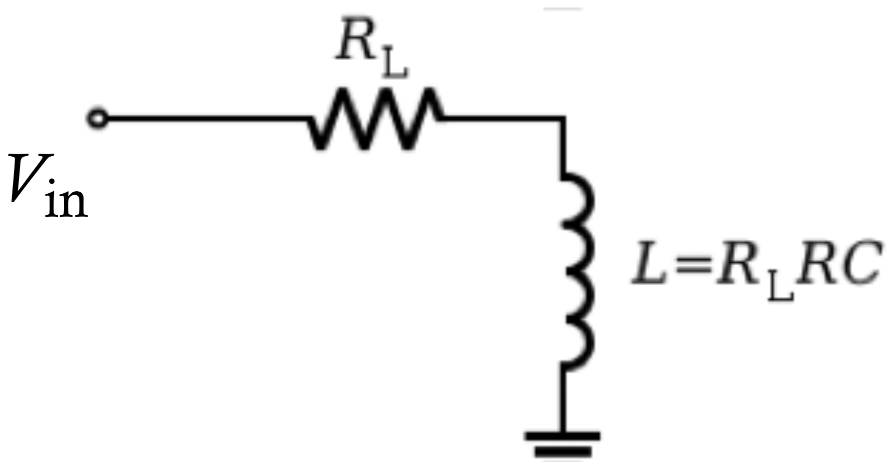


Inductance Gyrator



- Simulates an inductor
- Provides "inductance" without large, costly inductor

Equivalent Circuit



assume $R \gg R_L$

$$V_{out} = V_{in} - I_{in} R_L$$

$$V_{out} = V_{in} \frac{R}{R + j\omega C}$$

$$I_{in} R_L = \frac{V_{in}(\cancel{R} + 1/j\omega C - \cancel{R})}{R + 1/j\omega C}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{R_L(R + 1/j\omega C)}{1/j\omega C}$$

$$Z_{in} = j\omega \underbrace{R_L R C}_{\text{"L"}} + R_L$$

https://en.wikipedia.org/wiki/Gyrator