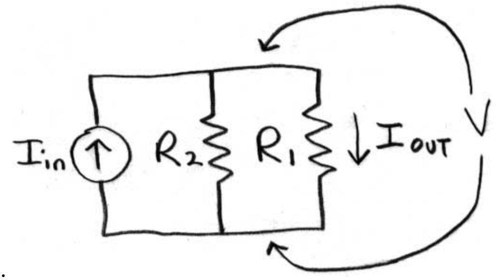
Current Divider

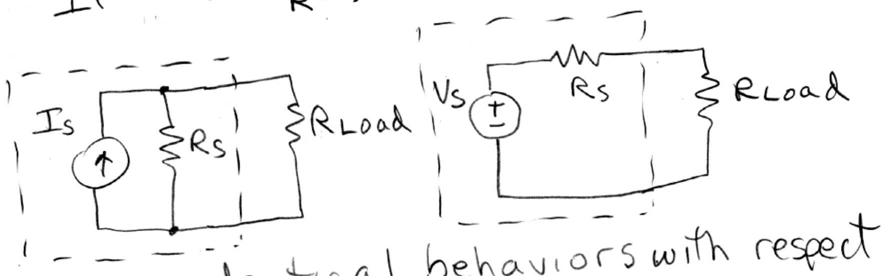
- View as “system” with current input and output.
- Same *voltage* across both resistors.
- Current through each resistor is proportional to its *conductance*.

$$I_{\text{in}} = V\left(G_1 + G_2\right)$$



$$I_{\text{out}} = VG_1 = \frac{R_2}{R_1 + R_2} I_{\text{in}}$$

If $I_s = \frac{V_s}{R}$, then the two models



have identical behaviors with respect to R_{Load}:

	R_{Load}	I_{Load}	V_{Load}
"short circuit"	0 Ω	I_s	0 V
"open circuit"	∞ Ω	0 A	V_s

called *Thevenin* and *Norton* equivalents

Thevenin Equivalent

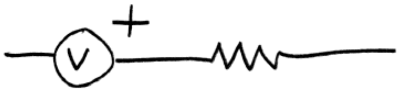
ANY NETWORK OF

 's,  I 's

and

V  's

can be redrawn as



Norton Equivalent

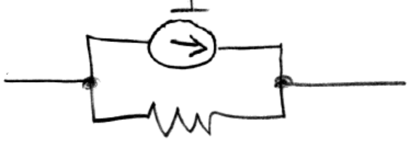
ANY NETWORK OF

 's,  I 's

and

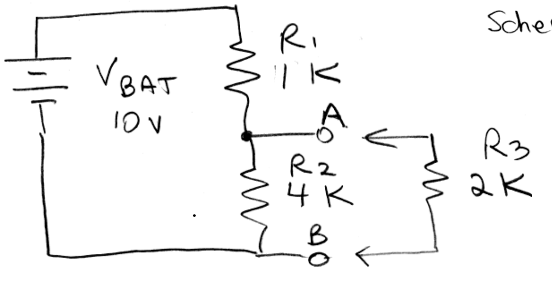
 V 's

Can be rewritten as



Thevenin Equivalent - Example

What is current through R_3 ?



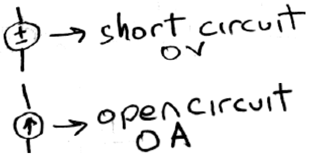
Scherz p. 77

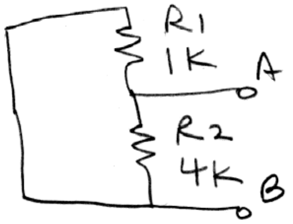
- Find V_{THEV} of "branch" (2-node component)
  - Compute voltage between A and B without R_3.

$$V_{\mathrm{THEV}} = V_{\mathrm{BAT}} \frac{4\mathrm{K}}{1\mathrm{K}+4\mathrm{K}} = 8\mathrm{V}$$

(continued….)

- Find R_{THEV} of branch
  - Set all sources to zero.

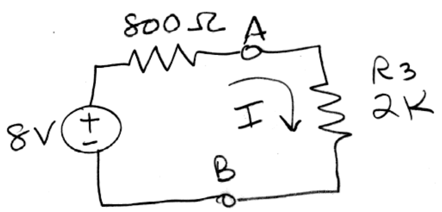




just parallel resistors

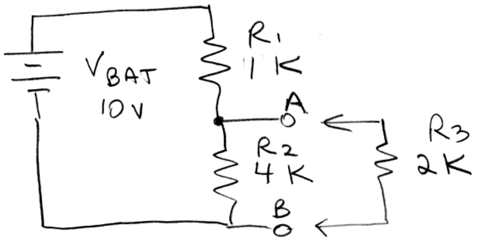
$$R_{\mathrm{THEV}} = \frac{R_1 R_2}{R_1+R_2} = 800\Omega$$

- Now use Thevenin Equivalent to find I through R_3.



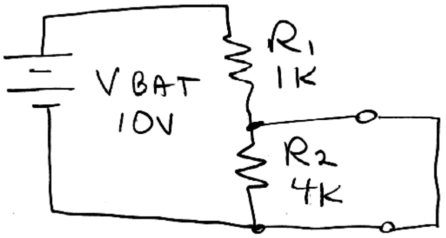
$$I = \frac{8\mathrm{V}}{2\mathrm{K}+800\Omega} \cong \boxed{3\mathrm{mA}}$$

Norton Equivalent - Example



Again, what is current through R_3 ?

- Find I_{NORT} of "branch" (2-node component)
  - Replace R_3 with short-circuit between A and B.

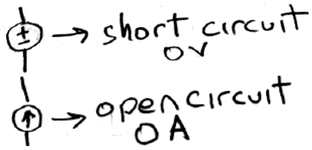


$$\downarrow I_{NORT} = \frac{10\text{V}}{1\text{K}} = 10\text{mA}$$

(continued….)

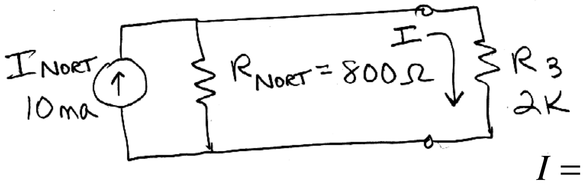
parallel resistors

- Find R_{NORT} the same way as before (R_{NORT}=R_{THEV})



$$R_{\text{THEV}} = \frac{R_1 R_2}{R_1 + R_2} = 800\Omega$$

- Now use Norton Equivalent to find I through R_3.



Use the current divider equation

$$I = I_{\text{NORT}} \frac{R_{\text{NORT}}}{R_3 + R_{\text{NORT}}} \cong \boxed{3\text{mA}}$$

Superposition Theorem

Superposition Theorem: The current in a branch = sum of currents produced by each source with the other sources set to 0

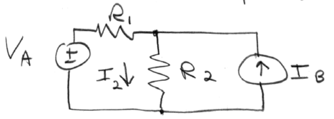
"set to 0" means

 → short circuit $V = 0$

 → open circuit $I = 0$

Superposition Theorem

Example (Scherz p75)

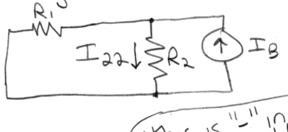


Break up $I_2 = I_{21} + I_{22}$

setting $I_B = 0$ (open circuit)

$$I_{21} = \frac{V_A}{R_1 + R_2}$$

Setting $V_A = 0$ (short circuit)



resistors in parallel have same voltage

$$I_{22} R_2 = \underbrace{(I_B - I_{22})}_{\text{current thru } R_1} R_1$$

$$I_{22} = \frac{I_B R_1}{R_1 + R_2}$$

This known as the "current divider eq."

This is "–" in book wrong!!

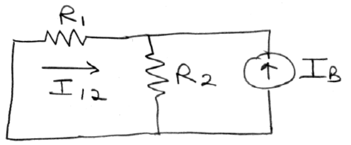
$$I_2 = I_{21} + I_{22} = \frac{V_A + I_B R_1}{R_1 + R_2}$$

Continuing the example, solve for I_1 thru R_1

setting $I_B = 0$

$$I_{11} = \frac{V_A}{R_1+R_2}$$

Setting $V_A = 0$



$$-I_{12}R_1 = \underbrace{(I_B+I_{12})}_{\text{current thru } R_2} R_2$$

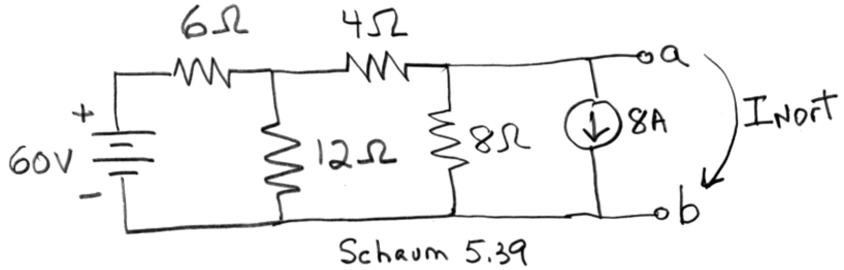
notice "–" because current is actually going the other way

$$I_{12} = \frac{-I_B R_2}{R_1+R_2}$$

$$I_1 = \frac{V_A - I_B R_2}{R_1+R_2}$$

Norton Equiv. by Superposition

Find Norton equivalent with I_{NORT} from *a* to *b.*

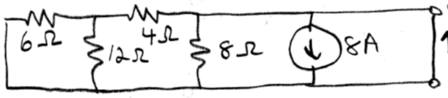


Schaum 5.39

Use superposition to find $\overline{I_{Nort}}$, the current from a to b with a shorted to b

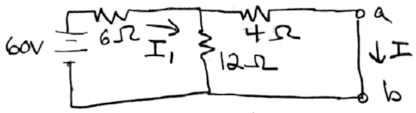
(continued….)

(1) Contribution from current source with battery at 0V (shorted). Ignore all resistors since all current goes through short from *a* to *b.*



↑ goes backwards $I = -8A$

(2) Contribution from battery with current source at 0A (open). Ignore 8 Ω resistor since it is parallel to short from *a* to *b.*



$$I_1 = \frac{60V}{6\Omega + \frac{12\Omega \cdot 4\Omega}{12\Omega + 4\Omega}} = 6.67A$$

↑ total current from battery fed into current divider

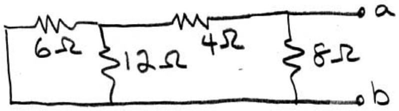
$$I_2 = I_1 \frac{12\Omega}{12\Omega + 4\Omega} = 5A$$

+

$$I_{Nort} = \boxed{-3A}$$

(continued….)

R_{Nort} is resistance between a and b with all sources 0.



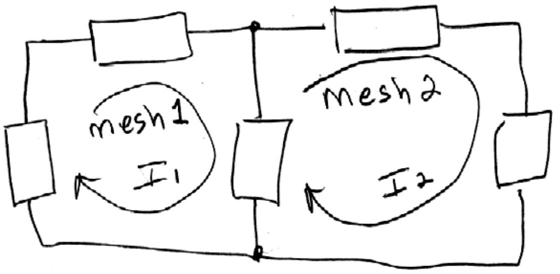
$$\frac{6\Omega\ 12\Omega}{6\Omega + 12\Omega} = 4\Omega$$

← simplify parallel resistors

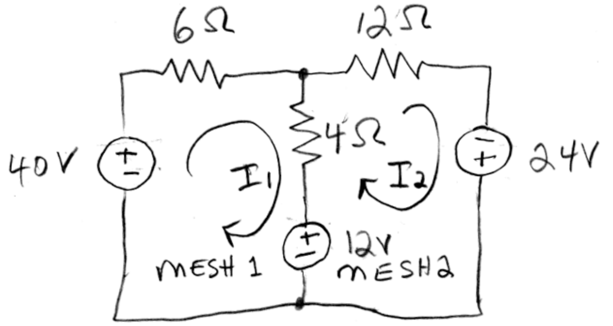
$$R_{Nort} = \frac{8\Omega \cdot 8\Omega}{8\Omega + 8\Omega} = \boxed{4\Omega}$$

Mesh analysis yield simultaneous linear equations.

mesh



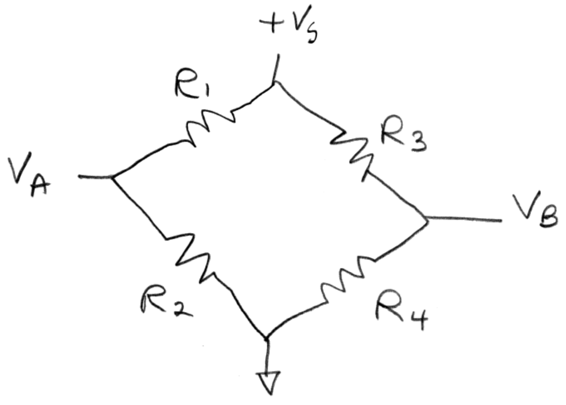
Example of a Mesh



USING KIRCHHOFF'S VOLTAGE LAW

$$10 I_1 - 4 I_2 = 28$$
$$-4 I_1 + 16 I_2 = 36$$

Wheatstone Bridge



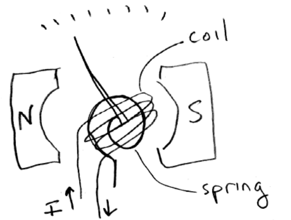
$V_A = V_B$ when $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

basically 2 voltage dividers

Wheatstone Bridge

accurate measurement setup independent of Vs

adjust pot so as to "zero" the voltmeter classic galvanometer Ⓖ



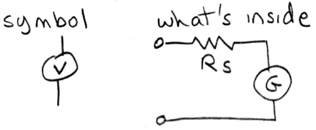
tiny current (50 μA)
Through coil
moves needle
Coil has ~0Ω resistance

your meter ↓

symbol

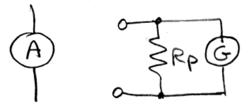
what's inside

voltmeter



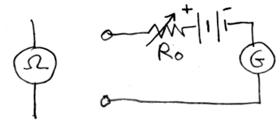
Rs is big, 1mΩ-1gΩ
("s" for series)

ammeter



Rp is small <1Ω
("p" for parallel
Easy to destroy meter!!

ohmmeter

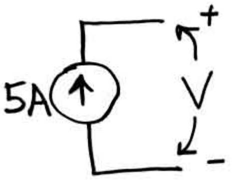


Ro used to adjust
offset.
injects current...
only use on isolated,
non-powered resistors.

Analog Volt-Ohm-Meter (VOM)

DC Circuits without solutions

- There are basically 2 illegal DC circuits
  - – Current source with open circuit: $V = \infty$



  - – Voltage source with short circuit: $I = \infty$