Homework 2 – For your own education, not to be handed in.

1. Answer the following questions about a theoretically perfect 4 H inductor.

A. If 0 V is applied for all $t < 0$, and 0 A is passing through it at $t = 0$, and then 2 V is applied across it from time $t = 0$ seconds to $t = 4$ seconds, calculate the current at $t = 4$ seconds.

B. What is the energy stored in the coil in question A at $t = 4$ seconds?

C. If a voltage $Asin(2\pi ft)$ is applied from $-\infty \leq t \leq +\infty$, $f = 100$ Hz, $A = 4$ V, derive a function for the current through the inductor.

D. What is the power *dissipated* by the inductor in question C? (note italics).

E. What voltage across the coil would theoretically be required to change the current through the coil at $t = 0$ from 0 A to 1 A, *instantaneously?*

2. Convert the following expressions to cartesian coordinates $z = x + jy$. Sketch the complex number as an arrow on the complex plane, indicating numerical values on the real and imaginary axes, angle, and radius.

A. $4e^{-j\frac{\pi}{4}}$

B. $e^{ln2+j\frac{\pi}{2}}$

C. $3e^{j\frac{11\pi}{4}}$

D. $\left(e^{j\frac{\pi}{2}}\right)^3$

3. Convert the following expressions to complex exponentials, in the form $re^{j\theta}$, where $r \geq 0$ and $-\pi < \theta \leq \pi$. Sketch the complex number as an arrow on the complex plane, indicating numerical values on the real and imaginary axes, angle, and radius

A. $\sqrt{3} - j$

B. $-2 + 2j$

C. $(1 - j)^3$ (hint: it's easier if you first convert $1 - j$ to a complex exponential and cube it, but you can do it both ways just to be sure).

D. $2ej$

4. Given the trigonometric identity $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

A. Rewrite the trigonometric identity, converting the trigonometric functions into their complex exponential equivalents.

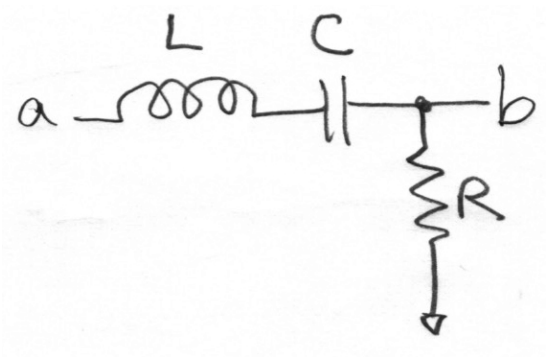
B. Show that the equation you have written in question A is, in fact, true.

5. Given the trigonometric identity $\sin 2\theta = 2\sin\theta cos\theta$

A. Rewrite the trigonometric identity, converting the trigonometric functions into their complex exponential equivalents.

B. Show that the equation you have written in question A is, in fact, true.

6. Given the following circuit,



A. Derive an equation for the total impedance Z_{ab} between points *a* and *b*, as a function of frequency ω.

B. What is Z_{ab} when $\omega = 0$? (include units)

C. What is Z_{ab} when $\omega = \infty$? (include units)

D. At what frequency ω_0 (in radians/second) does Z_{ab} = 0 ohms? (show your justification) What frequency f_0 in Hz does this correspond to?

E. Derive an equation for the transfer function $H(\omega)$ for the system whose input is at point *a* and output is at point *b*.

F. What will $H(\omega)$ be for $\omega = \omega_0$ from question D? Show your derivation.

G. If $cos(\omega_0 t)$ is presented at point *a*, what signal would you expect at point *b*? (Hint: Given your answer from question F, this should be trivial.)